MATH 174, ANSWERS TO PRACTICE PROBLEMS FOR TEST 1

1.

p	q	$p \wedge q$	$\sim p$	$\sim q$	$p \vee \sim q$	$\sim p \vee q$	$\sim p \vee (p \wedge q)$	$\sim q \vee (p \wedge q)$	$(p \vee \sim q) \vee (\sim p \vee q)$
T	T	T	F	F	T	T	T	T	T
T	F	F	F	T	T	F	F	T	T
F	T	F	T	F	F	T	T	F	T
F	F	F	T	T	T	T	T	T	T

2. $p \vee \sim q \equiv \sim q \vee (p \wedge q)$ and $\sim p \vee q \equiv \sim p \vee (p \wedge q)$
3. $(p \vee \sim q) \vee (\sim p \vee q)$
4. There are no contradictions.
5. $\sim p \wedge \sim q$
6. $\sim p \vee \sim q$
7. There are two variables, p and q, so the truth table would have 4 rows (like above). Thus, in each column, there are 4 rows of spaces each of which is either filled with "T" or "F". There are 16 different ways to fill the 4 spaces with T's and F's. It follows that if there are 17 or more columns (each representing a statment form), at least two of the columns must have the 4 spaces filled in exactly the same way. In other words, at least two of the statement forms in a collection of 17 or more forms would have to be equivalent.
8. if q then p (that is, $q \to p$)
9. if not p then not q (that is, $\sim p \to \sim q$)
10. p and not q (that is, $p \wedge \sim q$)
11. if not q then not p (that is, $\sim q \to \sim p$)
12. the contrapositive
13.

p	q	$p \wedge q$	$\sim q$	$\sim q \to p \wedge q$
T	T	T	F	T
T	F	F	T	F
F	T	F	F	T
F	F	F	T	F

14. The first two arguments are valid (the second two are invalid).
15. $\exists x \in D$ such that $\sim P(x)$
16. $\forall x \in D, \sim P(x)$
17. $\exists$ positive integers n such that $\forall$ integers $a, b, c,$ and $d, n \neq a^2 + b^2 + c^2 + d^2$
18. $\exists a \in \mathbb{Q}$ and $\exists b \in \mathbb{Q}$ such that $\forall c \in \mathbb{Q}$, $c \leq a$ or $c \geq b$.
19. $\sim Q(x) \implies P(x)$ and $P(x) \iff \sim Q(x)$
20. For $x = 0$, if $8x + 13y = 1$ then $y = 1/13$ so no *integer* y satisfies $8x + 13y = 1$ (in this case).
For $x = 1$, if $8x + 13y = 1$ then $y = -7/13$ so no integer y satisfies $8x + 13y = 1$ (in this case).
For $x = 2$, if $8x + 13y = 1$ then $y = -15/13$ so no integer y satisfies $8x + 13y = 1$ (in this case).
For $x = 3$, if $8x + 13y = 1$ then $y = -23/13$ so no integer y satisfies $8x + 13y = 1$ (in this case).
For $x = 4$, if $8x + 13y = 1$ then $y = -31/13$ so no integer y satisfies $8x + 13y = 1$ (in this case).
Thus, by the method of exhaustion, $\forall x \in \{0, 1, 2, 3, 4\}$, there does not exist an integer y such that $8x + 13y = 1$.
21. Since $8 \times 5 + 13 \times (-3) = 1$, there exist x and y such that $8x + 13y = 1$ (namely, $x = 5$ and $y = -3$ work).
22. 1, 2, 3, 4, 6, and 12
23. 3, 5, and 13

24. $2^4 \cdot 5$.

25. Yes. Since $9+8+7+6+5+4+3+2+1 = 45$ and 45 is divisible by 3, the number 987654321 is divisible by 3.

26. Since $200 \bmod 7 = 4$, it will be the same as four days from today. If you are reading this on Thursday, the answer is "Monday".

27. Yes. To check if 241 is prime, we need only see if 241 is divisible by a prime $\leq \sqrt{241}$. Since $16^2 = 256$, we need only consider dvisibility by primes < 16. One checks directly that 241 is not divisible by 2, 3, 5, 7, 11, and 13, which then justifies that 241 is prime.

28. $73 \bmod 5 = 3, \quad -73 \bmod 5 = 2, \quad 29 \bmod 4 = 1, \quad -29 \bmod 4 = 3$

29. $\lfloor 3.6 \rfloor = 3, \quad \lceil 3.6 \rceil = 4, \quad \lfloor -1.9 \rfloor = -2, \quad \lceil -1.9 \rceil = -1$

30. Assume $\sqrt{2}$ is rational. Then there exist $\boxed{\text{integers}}$ a and b with $b \neq 0$, with $\sqrt{2} = a/b$, and with a/b reduced (so that a and b have no common prime factors). Since $\sqrt{2} = a/b$, we obtain

$$b\sqrt{2} = a \qquad \text{so that} \qquad 2b^2 = a^2.$$

We deduce that a is even. Therefore, there is an integer k such that $\boxed{a = 2k}$. Substituting this into $2b^2 = a^2$, we obtain $\boxed{2b^2 = (2k)^2 = 4k^2}$ so that $b^2 = \boxed{2k^2}$. We deduce that $\boxed{b}$ is even. This is a contradiction since $\boxed{a/b \text{ is reduced and } a \text{ and } b \text{ are even}}$. Therefore, our assumption is wrong and $\sqrt{2}$ is irrational. ∎