MATH 174, LECTURE 9

1. Go over homework questions.

2. Homework: pages 146, numbers 1, 5, 7(b), 8(b), 13 (includes old problems)
   pages 153, numbers 1, 3, 10(a)
   Test: Tuesday (10/02)

Old

3. **Theorem 3.4.1** If n and d are integers with $d > 0$, then there exist unique integers q (called the "quotient") and r (called the "remainder") satisfying

$$n = dq + r \qquad \text{and} \qquad 0 \leq r < d.$$

   Note: The notation $n \bmod d$ is used to denote the remainder r.

New

4. **Examples:** (1) $15 \bmod 8 = 7$, $6 \bmod 6 = 0$, $100 \bmod 2 = 0$, $1234 \bmod 3 = 1$

   (2) $-2 \bmod 3 = 1$, $-5 \bmod 2 = 1$, $-23 \bmod 5 = 2$

   (3) $n \mod 4 \in \{0, 1, 2, 3\}$ (When is n odd?)

5. **Further Examples:** (1) Today is Thursday, 09/20. What day of the week is 09/20 on next year?

   (2) Squares mod 4 are 0 or 1.

   (3) Is 9830207487 a sum of two squares?

6. **Definitions and Notations:** The floor of x, denoted $\lfloor x \rfloor$, is the unique integer n such that $n \leq x < n + 1$. The ceiling of x, denoted $\lceil x \rceil$, is the unique integer n such that $n - 1 < x \leq n$.

   Comment: Sometimes the floor function is referred to as the greatest integer function and the notation $[x]$ is used.

7. **Examples:** (1) Compute $\lfloor x \rfloor$ and $\lceil x \rceil$ for $x \in \{3.7, 35/8, \sqrt{2}, \sqrt{119}, -3.2, -44/5, -0.999\}$.

   (2) If $n = qd + r$ as in Theorem 3.4.1, then write q and r using $\lfloor x \rfloor$ and mod notations.

   Proofs by Contradiction: Explain again and give the examples that $\sqrt{2}$ is irrational and that there are infinitely many primes.

8. Give quiz.