MATH 174, LECTURE 8

1. Return quizzes (33 total, 62.1% ave., 0 perfects, 6 scores of 9.5; 6 A's, 2 B's, 8 C's, 4 D's, 13 F's)

2. Go over homework questions.

3. Homework: pages 138–139, numbers 1, 3, 5, 6, 7, 11, 29, 30, 31, 33
   page 146, numbers 1, 5, 7(b), 8(b)
   Quiz: Thursday (09/20)
   Test: Tuesday (10/02) $\leftarrow$ as voted on in class

Old

4. **Definitions:** r is *rational* $\iff$ $\exists$ integers a and b such that $r = a/b$ and $b \neq 0$
   $r \in \mathbb{R}$ is *irrational* $\iff$ r is not rational

5. **Example:** Explain why the sum of two rational numbers is rational.

New

6. **Definitions and Notations:** For n and d integers with $d \neq 0$, we write $d|n$ (read "d divides n") if $\exists k \in \mathbb{Z}$ such that $n = kd$. The following all have the same meaning:

   d *divides* n
   d *is a factor of* n
   d *is a divisor of* n
   n *is divisible by* d
   n *is a multiple of* d

7. **Examples:** (1) Each of the following are true:

   $$2|22 \quad 3|21 \quad 7|21 \quad 6|(-36) \quad (-6)|36 \quad (-6)|(-36) \quad 1|17 \quad 23|0 \quad 3|(-3) \quad 6 \nmid 3 \quad 6 \nmid 7$$

   (2) What are the divisors of 18?

   (3) What are the prime divisors of 18?

   (4) If p is a prime, then what are its divisors?

   (5) page 138, numbers 4, 12

8. **Theorem 3.3.3** [Unique Factorization Theorem or The Fundamental Theorem of Arithmetic]: Every positive integer > 1 can be written uniquely in the form

   $$p_1^{e_1} p_2^{e_2} \cdots p_k^{e_k},$$

   where $p_1, p_2, \ldots, p_k$ are primes and $e_1, e_2, \ldots, e_k$ are positive integers, except for the order in which the prime powers appear.

9. **Examples:** (1) Completely factor 120?

   (2) Completely factor 221?

10. **Comment:** If a positive integer n is composite, then n has a divisor > 1 and $\leq \sqrt{n}$. Furthermore, one can find such a divisor that is prime.

11. **Theorem 3.4.1** If n and d are integers with $d > 0$, then there exist unique integers q (called the "quotient") and r (called the "remainder") satisfying

    $$n = dq + r \qquad \text{and} \qquad 0 \leq r < d.$$

    Note: The notation $n \mod d$ is used to denote the remainder r.