MATH 174, LECTURE 7

1. Go over homework questions.

2. Homework: pages 124–125, numbers 1, 3, 6, 7, 9, 15, 21, 25, 28, 29 (some from last time)
   Read from Example 3.1.5 ”Proving a Theorem” on page 117 to the middle of page 120

   Quiz: Thursday (09/20)

   Test: Thursday (09/27)

Old

3. **Definitions:** $n \in \mathbb{Z}$ is *even* $\iff$ $\exists k \in \mathbb{Z}$ such that $n = 2k$
   $n \in \mathbb{Z}$ is *odd* $\iff$ $\exists k \in \mathbb{Z}$ such that $n = 2k + 1$

4. **Definitions:**

   $n \in \mathbb{Z}$ with $n > 1$ is *prime* $\iff$ $\big(\forall$ positive integers r and s, $n = rs \implies$ either $r = 1$ or $s = 1\big)$
   $n \in \mathbb{Z}$ with $n > 1$ is *composite* $\iff$ $\exists$ integers $r > 1$ and $s > 1$ such that $n = rs$

5. **Constructive and Nonconstructive Proofs of Existence** (for existential statements)

   Examples: (1) There exist integers x and y such that $5x + 8y = 1$.
   (2) There exist numbers that are not rational. (Use $\sqrt{2}$ and $0.1010010001\ldots$)
   (3) There exist irrational numbers a and b such that a^b is rational.

6. **The Method of Exhaustion** (for universal statements)

   Examples: (1) The number 6174.
   (2) Every even number n with $4 \leq n \leq 30$ can be written as a sum of two primes.
   New (3) Every even number $n \geq 4$ can be written as a sum of two primes.

7. **Theorem 3.1.1** If the sum of two integers is even, then so is their difference.

   What does “two” mean here?

   Give the proof (note related to reading assignment).

8. **Common Errors:** (1) Arguing from examples (illustrate with 6174 and Theorem 3.1.1).
   (2) Using “if” inappropriately (Suppose $\sqrt{2}$ is rational. If $\sqrt{2}$ is rational, then $\sqrt{2} = a/b$ for some integers a and b.)

   Note that the students may want to look over other common errors on page 121.

9. **Disproofing a universal statement with a counterexample**

   Example: pages 124, number 16

10. **Miscellaneous Examples:** page 125, numbers 26, 37

11. **Definitions:** r is *rational* $\iff$ $\exists$ integers a and b such that $r = a/b$ and $b \neq 0$
    $r \in \mathbb{R}$ is *irrational* $\iff$ r is not rational

12. **Example:** Explain why the sum of two rational numbers is rational.

13. Give quiz.