MATH 174, LECTURE 6

1. Return quizzes (33 total, 85.3% ave., 10 perfects; 20 A's, 4 B's, 4 C's, 3 D's, 2 F's)

2. Go over homework questions.

3. Homework: pages 109–110, numbers 2, 3, 7, 8, 9, 10, 19(a&b), 21, 23 $\leftarrow$ validity not terminology
   page 124, numbers 1, 3, 6, 7, 9

   Quiz: Thursday (09/13)

4. **Different Types of Valid Arguments:**

   Universal Instantiation

   $\forall x \in D, P(x)$
   $a \in D$
   $\therefore P(a)$

   Universal Modus Ponens

   $\forall x \in D, P(x) \implies Q(x)$
   $P(a)$ for a particular $a \in D$
   $\therefore Q(a)$

   Universal Modus Tollens

   $\forall x \in D, P(x) \implies Q(x)$
   $\sim Q(a)$ for a particular $a \in D$
   $\therefore \sim P(a)$

5. **Different Types of Invalid Arguments:**

   Converse Error

   $\forall x \in D, P(x) \implies Q(x)$
   $Q(a)$ for a particular $a \in D$
   $\therefore P(a)$

   Inverse Error

   $\forall x \in D, P(x) \implies Q(x)$
   $\sim P(a)$ for a particular $a \in D$
   $\therefore \sim Q(a)$

6. **Examples:** pages 109–110, numbers 11, 12, 13, 15, 19(c)

7. **Diagram Examples:** page 110, numbers 24, 26

8. **Definitions:** $n \in \mathbb{Z}$ is *even* $\iff \exists k \in \mathbb{Z}$ such that $n = 2k$
   $n \in \mathbb{Z}$ is *odd* $\iff \exists k \in \mathbb{Z}$ such that $n = 2k + 1$

9. **Definitions:**

   $n \in \mathbb{Z}$ with $n > 1$ is *prime* $\iff$ $\big(\forall$ positive integers r and s, $n = rs \implies$ either $r = 1$ or $s = 1\big)$
   $n \in \mathbb{Z}$ with $n > 1$ is *composite* $\iff \exists$ integers $r > 1$ and $s > 1$ such that $n = rs$

10. **Constructive and Nonconstructive Proofs of Existence** (for existential statements)

    Examples: (1) There exist integers x and y such that $5x + 8y = 1$.
    (2) There exist numbers that are not rational. (Use $\sqrt{2}$ and $0.1010010001\ldots$.)
    (3) There exist irrational numbers a and b such that a^b is rational.

11. **The Method of Exhaustion** (for universal statements)

    Examples: (1) The number 6174.
    (2) Every even number n with $4 \leq n \leq 30$ can be written as a sum of two primes.