MATH 174, LECTURE 4

1. Return quizzes (35 total, 88% ave., 18 perfects; 21 A's, 6 B's, 3 C's, 2 D's, 3 F's)

2. Go over homework questions.

3. Homework: pages 87–88, numbers 1 (a-d), 4, 5, 6, 7, 9, 11 (a,c), 20 (a,c), 21 (a,b), 29, 32
   Quiz: Thursday (09/06)

Review

4. **Contradiction Rule:** If you can show that the supposition that p is false leads logically to a contradiction, then you can conclude that p is true.

5. **Examples:** (1) $\sqrt{2}$ is irrational
   (2) there exist irrational numbers α and β such that α^β is rational

New

6. **Definition:** A *predicate* is a sentence that contains a finite number of variables and becomes a statement when specific values are substituted for the variables. The *domain* of a predicate variable is the set of all values that may be substituted for the variable.

7. **Definition:** If $P(x)$ is a predicate and x has doemain D, the *truth set* of $P(x)$ is the set of elements of D that make $P(x)$ true when substituted for x.

   Note: This can be written $\{x \in D | P(x)\}$ (the set of all x in D such that $P(x)$ holds).

8. **Notation:** Let $P(x)$ and $Q(x)$ be predicates with domain D. The notation $P(x) \implies Q(x)$ means every element of the truth set of $P(x)$ is in the truth set of $Q(x)$. The notation $P(x) \iff Q(x)$ means $P(x)$ and $Q(x)$ have identical truth sets.

9. **Example:** (1) Suppose $P(x)$ is "x is even" and $Q(x)$ is "x is a power of 2" and $D = \{2, 3, \ldots, 100\}$.
   (2) Suppose further that $R(x)$ is "$x > 200$" (same D).
   (3) What if $D = \{2, 3, 4\}$?

10. **Definition and Notation:** The symbol "$\forall$" denotes "for all" and the symbol $\exists$ denotes "there exists". They are called *quantifiers*. The first is a *universal quantifier* and the second an *existential quantifier*.

11. $\forall x \in D, Q(x)$ (a *universal statement*)
    $\exists x \in D$ such that $Q(x)$ (an *existential statement*)

    When is each true?

    When is each false? (Note what a *counterexample* is.)

12. **Examples:** (1) Let $D = \{1, 2, \ldots, 10\}$. Then $\forall x \in D$, $x < 20$.
    (2) Let $D = \{1, 2, \ldots, 10\}$. Then $\forall x \in D$, $x > 5$.
    (3) Also, $\exists x \in D$ such that $x^2 \in D$ (same D).

13. Set Notations ($\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$, $\mathbb{C}$ or **Z**, **Q**, **R**, **C**)

    Examples: (1) $\exists x \in \mathbb{Z}$ such that $x^2 = x$ (write in English)
    (2) $\forall x \in \mathbb{R}, x^2 > x$ (not true)

14. **Negations:** $\sim (\forall x \in D, Q(x)) \equiv \exists x \in D$ such that $\sim Q(x)$
    $\sim (\exists x \in D$ such that $Q(x)) \equiv \forall x \in D, \sim Q(x)$

    Note: The negation of a universal statement is logically equivalent to an existential statement. The negation of an existential statement is logically equivalent to a universal statement.

    Examples: pages 87–88, numbers 11 (b,d), 20 (b,d)