# MATH 174, LECTURE 20

1. Go over homework.

2. Homework: pages 616–617, numbers 5, 7, 15, 16,19, 22

3. **Definitions:** A *graph* $G$ consists of two finite sets: a set $V(G)$ of *vertices* and a set $E(G)$ of *edges*. Each edge is associated with a set of *endpoints*, either one vertex (in the case of a *loop*) or two vertices. Two vertices connected by an edge are called *adjacent*. The *degree* of a vertex $v$ is the number of edges that have $v$ as an endpoint with a loop counting twice. The *total degree* of $G$ is the sum of the degrees of its vertices. A simple graph is a graph without loops and without any two edges associated with the same two points.

4. **Examples:** (1) Give an example of a graph. Label the edges and vertices. Write out $V(G)$ and $E(G)$. Indicate the vertices associated with each edge. What are the endpoints of the edges? Which vertices are adjacent? What are the degrees of the vertices? What is the total degree of the graph?

   (2) Add a loop to the graph and discuss changes to the above.

   (3) Add an edge with the same vertices as another edge and discuss changes to the above.

   (4) Discuss a star shaped graph and a pentagon shaped graph. Are they the "same" graph?

   (5) page 616, number 6

   (6) page 618, number 28 (explain why the total degree is the same as twice the number of edges)

5. **Definitions:** A *complete graph on $n$ vertices*, denoted $K_n$, is a simple graph on $n$ vertices that has for each pair of vertices an edge associated with them. A *complete bipartite graph on $(m, n)$ vertices*, denoted $K_{m,n}$, that contains a vertex set consisting of $m + n$ vertices $v_1, \ldots, v_m$ and $w_1, \ldots, w_n$ satisfying:

   (i) There is an edge associated with each pair of vertices $v_i$ and $w_j$.

   (ii) There are no edges associated with a pair $v_i$ and $v_j$.

   (iii) There are no edges associated with a pair $w_i$ and $w_j$.

6. **Examples:** (1) Illustrate $K_n$ for $n \in \{1, 2, \ldots, 5\}$. How many edges does $K_n$ have?

   (2) Illustrate $K_{2,3}$, $K_{1,3}$, $K_{3,3}$, and $K_{2,2}$.

7. **Definition:** A graph $G$ is planar if it can be represented as above (in a plane) without any edges crossing (if two edges have a common point, then that common point is an endpoint for both edges).

8. **Examples:** (1) Note the fourth example above.

   (2) Does a square with diagonals drawn represent a planar graph?

   (3) What if a roof is on top?

   (4) Is $K_5$ planar?

   (5) Discuss the $K_{3,3}$ problem.

   (6) Describe what graphs are planar in general.