MATH 174, LECTURE 2

1. Go over homework questions.
2. Homework: pages 27–28, numbers 2, 3, 4, 5, 9, 15, 16 (a-e), 20, 21, 26, 34, 36, 38
   Quiz: Thursday (08/30)
3. **Definition:** The statement "if p then q" or "p implies q" (denoted $p \to q$) is called *conditional*, with *hypothesis* p and *conclusion* q.

   When is $p \to q$ true? (For what truth values of p and q?)

   Draw truth table.
4. $p \to q \equiv \sim p \vee q$

   When is $\sim p \vee q$ true? (For what truth values of p and q?)

   Draw truth table to establish equivalence.
5. $p \to q \equiv \sim p \vee q \equiv \sim (\sim q) \vee \sim p \equiv \sim q \to \sim p$

   Definition: The *contrapositive* of "if p then q" is "if not q then not p".

   Conclusion: A conditional statement is logically equivalent to its contrapositive.
6. Recall De Morgan's Law $\sim (p \vee q) \equiv \sim p \wedge \sim q$.

   What does De Morgan's Law imply about $\sim (\sim p \vee q)$?

   Deduce $\sim (p \to q) \equiv p \wedge \sim q$.

   Example: Problem 16 (f) on page 27.
7. **Definitions:** The *converse* of "if p then q" is "if q then p". The *inverse* of "if p then q" is "if not p then not q".

   Comments: (1) The converse and the inverse of a conditional statement are logically equivalent to each other.
   (2) Neither is logically equivalent to the condtional statement.

   Example:
   - If I've eaten an apple, then I've eaten a fruit.
   - If I've eaten a fruit, then I've eaten an apple.
   - If I've not eaten an apple, then I've not eaten a fruit.
   - If I've not eaten a fruit, then I've not eaten an apple.
8. **Definition:** The statement "p *only if* q" means p is not true if q is not true.

   Comment: p only if q is not the same as p if q

   p only if q means $\sim q \to \sim p \equiv p \to q$
9. **Definition:** The *biconditional of* p and q is "p if and only if q" (denoted $p \leftrightarrow q$ and written sometimes p iff q). It is true precisely when p and q have the same truth values.

   Draw truth table.

   $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$
10. Order of operations ($\sim$ before $\wedge$ and $\vee$ before $\to$ and $\leftrightarrow$)
11. **Definitions:** The statements "p *is a sufficient condition for* q" and "q *is a necessary condition for* p" mean the same as "if p then q".

    Comment: p is a necessary and sufficient condition for q means the same as p iff q.