MATH 174, LECTURE 19

1. Return tests (30 total, plus 8, 80% ave., high 104.5; 4 A's, 12 B's, 10 C's, 3 D's, 1 F)
   Grades Before Final (2 quizzes dropped, high 97.2%; 7 A's, 16 B's, 2 C's, 3 D's, 3 unknowns)
   Final Comments: 50% if it helps

2. Go over homework.

3. Homework: pages 493–494, numbers 9, 11, 13, 25
   Read over examples from class.

4. **Definition and Notation:** The function $f(x)$ is big-oh of the function $g(x)$ provided there are real numbers x_0 and M such that
$$|f(x)| \leq M|g(x)| \qquad \text{for } x \geq x_0.$$

5. **Examples:** (1) $1/x = O(1)$

   (2) $(x-1)^2 = O(x^2)$

   (3) $(x+1)^2 = O(x^2)$ (note that $x+1 \leq 2x$ for $x \geq 1$)

   (4) $20(x+12)^4 = O(x^4)$

   (5) $20(x+12)^4 = O(x^{12})$

   (6) $f(x) = O(x^n)$ if $f(x)$ is a polynomial of degree n

   (7) $\sum_{k=1}^{n} k^r = O(n^{r+1})$

6. **Some Inequalities:**
   - $k+1 \leq 2^{k-1}$ for $k \geq 3$ (by induction)
   - $x < 2^{x/2}$ for $x > 8$ (consider $k \geq 3$ such that $2^k < x \leq 2^{k+1}$ and use the above)
   - $C(x+1) < 2^x$ for $x \geq x_1$ (take x_1 so that $2^{x/2} > 2C$ and then use $C(x+1) < C(x+x) = 2Cx < 2^{x/2}2^{x/2} = 2^x$)
   - $x^C < 2^x$ for $x \geq x_2$ (for $k > x_1$ and $2^k < x \leq 2^{k+1}$, $x^C < 2^{C(k+1)} < 2^{2^k} < 2^x$)
   - $f(x) = O(a^x)$ whenever $f(x)$ is a polynomial and $a > 1$
   - $(\log_2 x)^C = O(x)$ (let $t = x^{1/C}$ and $t^C < 2^t$ for t large)

7. **Further Examples:** (1) page 493, number 10

   (2) page 493, number 12