MATH 174, LECTURE 18

1. Return quizzes (25 total, 13.28% ave.; 23 A's, 2 B's)

2. Go over homework.

3. Homework: page 438, numbers 1, 3, 5
   page 451, numbers 3, 5, 10
   Solve $u_0 = 4$, $u_1 = -1$, and $u_{n+1} = u_n + 2u_{n-1}$ for $n \geq 1$.

4. **Recursion Examples:** (1) The Tower of Hanoi (show that $a_{n+1} = 2a_n + 1$ and $a_n = 2^n - 1$)

   (2) Maximal number of regions divided by n lines (show $a_{n+1} = a_n + n + 1$)

   (3) Fibonacci Numbers ($f_1 = 1$, $f_2 = 1$, and $f_{n+1} = f_n + f_{n-1}$ for $n \geq 2$)

5. **Solving Linear Recurrence Relations:** Given u_0, u_1, and $u_{n+1} = au_n + bu_{n-1}$ for $n \geq 1$, there exist numbers A and B such that
   $$u_n = A\alpha^n + B\beta^n$$
   where α and β are the roots of $x^2 - ax - b$ (provided $\alpha \neq \beta$). The numbers A and B are determined by the values of u_0 and u_1.

6. **Examples:** (1) Solve $u_0 = 1$, $u_1 = 3$, and $u_{n+1} = 3u_n - 2u_{n-1}$ for $n \geq 1$.

   (2) Solve $u_0 = 3$, $u_1 = 1$, and $u_{n+1} = 2u_n + 3u_{n-1}$ for $n \geq 1$.

   (3) Solve for f_n (the n^{th} Fibonacci number).

7. **Other Recursion Examples:** (1) page 438, number 2

   (2) page 438, number 4

   (3) page 451, number 4

   (4) page 451, number 6

8. **A Final Example:** Solve the linear recurrence $u_0 = 6$, $u_1 = 1$, $u_2 = 9$, and $u_{n+1} = 2u_n + u_{n-1} - 2u_{n-2}$ for $n \geq 2$. The sequence begins
   $$6,\ 1,\ 9,\ 7,\ 21,\ 31,\ 69,\ 127,\ 261,\ 511,\ 1029,\ 2047.$$