MATH 174, LECTURE 13

1. Go over homework questions.
2. Homework: pages 256–257, numbers 1, 2(a), $\underbrace{\text{11, 13, 15, 18, 25(a)}}_{\text{true or false, use Venn diagrams}}$

   pages 266–267, numbers 1, 6(a), 20(a), 36, 38

   Quiz: Thursday (10/25)
3. **Theorem 5.2.1:** For all sets A, B, and C,the following are true:
   1. $A \cap B \subseteq A$ and $A \cap B \subseteq B$.
   2. $A \subseteq A \cup B$ and $B \subseteq A \cup B$.
   3. if $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.
4. **Theorem 5.2.2:** Let A, B, and C be subsets of a universal set U. Then
   1. $A \cap B = B \cap A$ and $A \cup B = B \cup A$ (cummutative law)
   2. $A \cap (B \cap C) = (A \cap B) \cap C$ and $A \cup (B \cup C) = (A \cup B) \cup C$ (associative law)
   3. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ and $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ (distributive law)
   5. $(A^c)^c = A$ (double complement law)
   7. $(A \cup B)^c = A^c \cap B^c$ and $(A \cap B)^c = A^c \cup B^c$ (De Morgan's Laws)
   10. $A - B = A \cap B^c$ (alternative representation of set difference)
5. **Notes on Theorem 5.2.2:**
   2. Illustrate with a Venn Diagram. Note that we can deduce that we do not need parentheses here.
   3. Prove this different from book with following steps:
      - What does it mean for the sets to be equal? What does the book do?
      - Does this look familiar? Recall that $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$. Note that one is true if and only if the other one is true.
      - Let p be the statement $x \in A$, q the statement $x \in B$, and r the statement $x \in C$. Note that $q \wedge r$ is the same as $x \in B \cap C$ and $p \vee (q \wedge r)$ is the same as $x \in A \cup (B \cap C)$.
      - What is $p \vee q$? What about $p \vee r$? What about $(p \vee q) \wedge (p \vee r)$?
      - Tie the above together.
   5. Illustrate with a Venn Diagram.
   7. How would we prove this as above? Use that $\sim (p \vee q) \equiv \sim p \wedge \sim q$ where p is that $x \in A$ and q is that $x \in B$.
   10. Illustrate with a Venn Diagram.
6. **Examples:** (1) page 257, number 14

   (2) page 257, number 16
7. **Theorem 5.3.3:** Let A be a subset of a universal set U. Then

   1 & 3. $A \cup \emptyset = A$ and $A \cap \emptyset = \emptyset$

   2. $A \cap A^c = \emptyset$ and $A \cup A^c = U$
   4. $U^c = \emptyset$ and $\emptyset^c = U$
8. **Definition:** Two sets A and B are *disjoint* if $A \cap B = \emptyset$. More than two sets are said to be *mutually disjoint* (or *pairwise disjoint*) if no two of them share a common element (i.e., $A_1, \ldots, A_n$ are mutually disjoint if for all i and j with $1 \leq i < j \leq n$, $A_i \cap A_j = \emptyset$).

9. **Definition:** A collection of nonempty sets $\{A_1, \ldots, A_n\}$ is a *partition* of a set A if the sets are mutually disjoint and $A = A_1 \cup \ldots \cup A_n$.

10. **Example:** The sets $\{1, 4, 5\}$ and $\{2, 3, 6\}$ form a partition of the integers from 1 to 6 (inclusive).

11. **Definition:** Given a set A, the *power set* of A, denoted $\mathcal{P}(A)$, is the set of all subsets of A.

12. **Examples:** (1) If $A = \{0, 1\}$, then what is $\mathcal{P}(A)$?

    (2) If $A = \{0, 1, 2\}$, then what is $\mathcal{P}(A)$?

    (3) If $A = \{0\}$, then what is $\mathcal{P}(A)$?

    (4) If $A = \{\}$, then what is $\mathcal{P}(A)$?

    (5) If A is a set with n elements, how many elements does $\mathcal{P}(A)$ have? Why?

13. **Comment:** If $A \subseteq B$, then $\mathcal{P}(A) \subseteq \mathcal{P}(B)$.

14. Give quiz.