MATH 174, LECTURE 12

1. Go over homework questions.
2. Homework: pages 242–243, numbers 1, 2, 5, 6, 7, 8, 11(a), 13, 15(a,c,e,f), 17(a,d), 18(a)
   Quiz: Thursday (10/11)
3. **Sets and Elements of Sets:** Strictly speaking *sets* and *elements* are undefined terms. Think of a set as a collection of its elements.
4. **Notation via Examples:** (1) $\{-1, 2, 5\}$ is the set consisting of the elements -1, 2, and 5

   (2) {Bill, giraffe, book} is the set consisting of the elements Bill, giraffe, and book

   (3) $\{\,\}$ is the set consisting of no elements and is called the empty set (also denoted $\emptyset$)

   (4) $\{1, \{1\}\}$ is the set consisting of the two elements 1 and the set $\{1\}$

   (5) $\{x \in \mathbb{Z} \,|\, 1 \leq x \leq 10\}$ (the set of x in $\mathbb{Z}$ such that ...) is the set consisting of the integers from 1 to 10

   (6) What is $\{x \,|\, x \in \mathbb{Z}, 1 \leq x \leq 10\}$?

   (7) What is $\{x \in \mathbb{Z} \,|\, 2 \leq x^4 \leq 10\}$?
5. **Definition and Notation:** For two sets A and B, the notation $A \subseteq B$ means $\forall x,$ if $x \in A,$ then $x \in B$. The notation $A \subseteq B$ can be read in any of the following ways:

   A is a subset of B
   A is contained in B
   B contains A
6. **Question:** What would A is not a subset of B mean?

   Answer: $A \not\subseteq B \iff \exists x \in A$ such that $x \notin B$
7. **Definition:** For two sets A and B, we say A *is a propoer subset of* B if $A \subseteq B$ and there is at least one element of B not in A.
8. **Definition:** Two sets A and B are *equal* if $A \subseteq B$ and $B \subseteq A$.
9. **Definitions and Notations:** Let A and B be subsets of a "universal" set U. Then

   $A \cup B = \{x \,|\, x \in A$ or $x \in B\}$ ("A union B")
   $A \cap B = \{x \,|\, x \in A$ and $x \in B\}$ ("A intersect B")
   $A - B = \{x \,|\, x \in A$ and $x \notin B\}$ ("A minus B")
   $A^c = \{x \in U \,|\, x \notin A\}$ ("A complement")
   $A \times B = \{(a, b) \,|\, a \in A$ and $b \in B\}$ ("Cartesian product of A and B")

   Comments: Ordered tuples (a, b) as above are like points. Two tuples (a, b) and (c, d) are equal if and only if $a = c$ and $b = d$. Also, the notions of simultaneously taking the unioin, intersection, or Cartesian product of more than two sets has a natural generalization of the above.
10. **Examples:** (1) $\mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$

    (2) $A \cap B \subseteq A \cup B$

    (3) For $A = \{x \in \mathbb{Z} \,|\, 1 \leq x \leq 4\}$ and $B = \{x \in \mathbb{Z} \,|\, 3 \leq x \leq 7\}$, compute $A \cup B$, $A \cap B$, $A - B$, $B - A$, $A \times B$ and A^c if $U = \mathbb{Z}$.

    (4) page 243, number 9

    (5) page 243, number 15(b,d) (Venn diagrams)