MATH 174, LECTURE 11

1. Return tests (32 total, 81.17% ave.; 7 A's, 13 B's, 5 C's, 6 D's, 1 F)

2. Hand out solutions.

3. Go over homework questions.

4. Homework: pages 204–205, numbers 1, 9, 19, 20, 22, 24
   Quiz: Thursday (10/11)

5. **The Principle of Mathematical Induction:** Let $P(n)$ be a predicate defined for integers n. Let a be a fixed integer. Suppose the following hold:

   - $P(a)$ is true.
   - $\forall k \in \mathbb{Z}$ with $k \geq a$, if $P(k)$ is true, then $P(k+1)$ is true.

   Then $P(n)$ is true for all integers $n \geq a$.

   Comments: (1) Falling dominoes can be used to illustrate the idea.

   (2) This is another proving technique (like "proofs by contradiction").

6. **Examples:** (1) The first n positive consecutive odd numbers sum to n^2. (Give two proofs.)

   (2) The first n positive integers sum to $n(n+1)/2$. (Give two proofs. Note Gauss story.)

   (3) Let $a_1 = \sqrt{2}$ and $a_{n+1} = \sqrt{2 + a_n}$ for $n \geq 1$. Explain why $a_n \leq 2$ for all n.

   (4) Problem 2 on page 204.

7. **Theorem 4.2.3 (Sum of a Geometric Sequence):** For every real number $r \neq 1$ and every integer $n \geq 0$,

$$\sum_{j=0}^{n} r^j = \frac{r^{n+1} - 1}{r - 1}.$$

   Comment: Give three proofs (by induction, by considering rS where S is the above sum, and by expanding $r - 1$ times the sum).

8. **Examples:** (1) Compute $\sum_{j=0}^{10} 1/3^j$.
   (2) Compute $\sum_{j=0}^{10} (-1)^j/3^j$.
   (3) Compute $\sum_{j=10}^{100} j$.

9. Begin basic discussion of sets (equality of sets, containment, union, intersection).