MATH 174, LECTURE 10

1. Return quizzes (28 total, 113.9% ave.; 24 A's, 1 B, 1 C, 1 D, 1 F)
   (Note: of the 28 students taking the quiz, 15 currently have A's for their total quiz grades.)

2. Go over homework questions.

3. Handout practice test problems.

4. Homework: pages 153, numbers 1, 3, 10(a) (old problems)
   practice test problems
   pages 192–193, numbers 1, 3, 5, 10, 11, 12, 19, 24, 29, 35, 37 (due 10/04)
   Test: Tuesday (10/02)

Old

5. **Definitions and Notations:** The floor of x, denoted $\lfloor x \rfloor$, is the unique integer n such that $n \leq x < n+1$. The ceiling of x, denoted $\lceil x \rceil$, is the unique integer n such that $n-1 < x \leq n$.

   Comment: Sometimes the floor function is referred to as the greatest integer function and the notation $[x]$ is used.

6. **Examples:** (1) Compute $\lfloor x \rfloor$ and $\lceil x \rceil$ for $x \in \{3.7, 35/8, \sqrt{2}, \sqrt{119}, -3.2, -44/5, -0.999\}$.

   (2) If $n = qd + r$ as in Theorem 3.4.1, then write q and r using $\lfloor x \rfloor$ and mod notations.

7. **Proofs by Contradiction:** Explain again and give the examples that $\sqrt{2}$ is irrational and that there are infinitely many primes.

New

8. **Definitions:** A *sequence* $a_m, a_{m+1}, a_{m+2}, \ldots, a_n$ is an ordered list consisting of an *initial term* a_m and sometimes a *final term* a_n. If there is no final term, the sequence $a_m, a_{m+1}, a_{m+2}, \ldots$ is said to be an *infinite sequence*.

9. **Examples:** (1) Consider $a_1, a_2, \ldots$ given by $1, -1, 1, -1, \ldots$. What's a_k?
   (2) Consider $a_0, a_1, \ldots$ given by $1, -1, 1, -1, \ldots$. What's a_k?
   (3) Consider $a_0, a_1, \ldots$ given by $1, 2, 3, 4, \ldots$. What's a_j?
   (4) If $b_n = \lfloor n/3 \rfloor$, what are the first few terms of the sequence $b_1, b_2, \ldots$?
   (5) Find an "explicit formula" for the sequence $1, 1/2, 1/4, \ldots$.

10. **Notation:** Let n and m be (nonnegative) integers with $m \leq n$. The notation $\sum_{k=m}^{n} a_k$ denotes the sum of the elements of the sequence $a_m, a_{m+1}, \ldots, a_n$. In other words,

$$\sum_{k=m}^{n} a_k = a_m + a_{m+1} + \cdots + a_n.$$

    In the case that $m > n$, the sum is said to be (vacuously) 0.

11. **Examples:** (1) Compute $\sum_{k=1}^{5} k$ and $\sum_{j=0}^{5} j$.

    (2) Compute $\sum_{k=1}^{5} 1$ and $\sum_{j=0}^{5} 1$.

    (3) Compute $\sum_{n=1}^{5} ((n+1)^2 - n^2)$.

    (4) How would you write $1 + 2 + 2^2 + \cdots + 2^n$ using summation notation?

12. **Definition and Notation:** The product of the first n positive integers is n *factorial* and is denoted by $n!$. Also, we define $0!$ to be 1.

    Examples: $3! = 6$, $5! = 120$, $10! = 3628800$, $30! = 265252859812191058636308480000000$