MATH 174, LECTURE 1

1. Pass out and discuss course syllabus.
   - classroom changes on Tuesday (for computer use)
   - first test may not be before drop day
2. class photos
3. Homework: pages 15-16, numbers 1, 3, 6, 8, 12, 14, 17, 18, 20, 23, 24, 27, 29, 38

   Note: Not collected. Ask questions in class.
4. Quiz: Next Thursday (on above homework).
5. Topics for course (logic, number theory, proof techniques, combinatorics)
6. Logic has to do with "form" rather than "content". Do problem 2, page 15. Emphasize use of variables.
7. **Definition:** A *statement* is a sentence which is either true or false but not both.

   Examples: (1) Six is one less than five.
   (2) Six is one more than five.

   Non-Examples: (1) $x > 5$
   (2) The boy is in this classroom.
8. **Definition:** If p is a statement variable, the *negation* of p is "not p" (denoted $\sim p$).

   Example: The negation of "cold" is "not cold" (note this counts lukewarm; also we have substituted a value for the variable).
9. **Definition:** If p and q are statement variables, the *conjunction* of p and q is "p and q" (denoted $p \wedge q$).

   Example: The conjunction of $x > 0$ and $x \leq 2$ is $0 < x \leq 2$.

   Question: What does it take for $p \wedge q$ to be true?
10. **Definition:** If p and q are statement variables, the *disjunction* of p and q is "p or q" (denoted $p \vee q$).

    Examples and Non-Examples: (1) "Do you want cream or sugar with your coffee?" (inclusive)
    (2) "Do you want coffee, tea, or milk?" (exclusive (?))

    Question: What does it take for $p \vee q$ to be true?
11. **Definition:** A *statement form* is an expression made up of statement variables (like p and q) and logical connectives (like $\sim$, $\wedge$, and $\vee$) that becomes a statement when actual statements are substituted for the variables.

    Examples and Non-Examples: (1) $\sim (p \wedge q)$
    (2) $\sim p \wedge \sim q$
12. Order of operations: $\sim$ before $\wedge$ and $\vee$
13. Which of these mean the same thing? $\sim (p \wedge q)$, $\sim (p \vee q)$, $\sim p \wedge \sim q$, $\sim p \vee \sim q$

    What does that last question mean?

    Definition: Two statement forms P and Q are *logically equivalent* if they have identical truth values for each possible substitution of statements for their statement variables (denoted $P \equiv Q$).
14. Truth Tables. Illustrate with $\sim (p \wedge q)$, $\sim (p \vee q)$, $\sim p \wedge \sim q$, $\sim p \vee \sim q$ (discuss answer to previous questions), $p \wedge \sim p$, and $p \vee \sim p$.

15. De Morgan's Laws: $\sim (p \wedge q) \equiv \sim p \vee \sim q$
$\sim (p \vee q) \equiv \sim p \wedge \sim q$

16. **Definitions:** A *tautology* is a statement form that is always true regardless of the truth values of the statement variables. A *contradiction* is a statement form that is always false regardless of the truth values of the statement variables.

    Examples: See above truth tables.

17. Theorem 1.1.1 on Logical Equivalences. Let p, q, and r be variables, t a tautology, and c a contradiction. Then (for example)

    - $p \wedge q \equiv q \wedge p$
    - $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
    - $p \wedge t \equiv p$
    - $p \wedge \sim p \equiv c$
    - $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
    - $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

18. Problems for Further Discussion: pages 15–16, numbers 7 (use of "but"), 28 (De Morgan's Laws)