MATH 141: TEST 3

Name _____

Instructions and Point Values: Put your name in the space provided above. Check that your test has exactly 6 different pages including one blank page. Work each problem below and show ALL of your work. You do not need to simplify your answers. Do NOT use a calculator.

Problem (1) is worth 10 points.
Problem (2) is worth 20 points.
Problem (3) is worth 14 points.
Problem (4) is worth 14 points.
Problem (5) is worth 14 points.
Problem (6) is worth 14 points.
Problem (7) is worth 14 points.

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(1) Given that $\int_0^3 f(x)\,dx = 1$, $\int_0^4 f(x)\,dx = 6$, and $\int_1^3 f(x)\,dx = 2$, calculate $\int_1^4 f(x)\,dx$.

(2) Calculate each of the following integrals.

(a) $\displaystyle\int_0^{\pi} \sin\theta \, (\cos\theta)^{10} \, d\theta$

(b) $\displaystyle\int \frac{t \, dt}{\sqrt{t^2+1}}$

(c) $\displaystyle\int x(2x-1)^{5/2} \, dx$

(d) $\displaystyle\int x\sqrt{x} \, dx$

(3) Given that $F(x) = \int_x^{x^2} \sqrt{t^3 + 3}\, dt$, calculate $F(1)$, $F'(x)$, and $F'(1)$. (Show work.)

$F(1) =$ $\boxed{}$

$F'(x) =$ $\boxed{}$

$F'(1) =$ $\boxed{}$

(4) Calculate the area of the region bounded by the graphs of $x = y^2 - y$ and $x = y - y^2$.

(5) The region below the graph of $y = \sqrt{\sin x}$ and above the $x-$axis between $x = 0$ and $x = \pi$ revolves about the $x-$axis to form a solid. Calculate the volume of the solid. Your answer should be a number (i.e., set up an integral AND evaluate it).

(6) The region below the graph of $y = \sqrt{\sin x}$ and above the $x-$axis between $x = 0$ and $x = \pi$ (the same region as in problem (5) above) revolves about the $y-$axis to form a solid. Express the volume of the solid as an integral, but do NOT evaluate the integral.

(7) Calculate the integral $\int_a^b f(x)\,dx$ boxed below in the following way. Divide the interval $[a, b]$ into n equal subintervals, calculate the area of the corresponding circumscribed polygon, and then let $n \to \infty$. You should make use of the formula

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}.$$

Your final answer should be a number.

$$\boxed{\int_0^2 (2x+3)\,dx}$$