MATH 141: TEST 2

Name _____

Instructions and Point Values: Put your name in the space provided above. Check that you have 6 (different) test pages. Work each problem below and show ALL of your work. Unless specified otherwise, you do not need to simplify your answers. Do NOT use a calculator.

Problem (1) is worth 12 points.
Problem (2) is worth 12 points.
Problem (3) is worth 14 points.
Problem (4) is worth 18 points.
Problem (5) is worth 30 points.
Problem (6) is worth 14 points.

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(1) (a) Calculate $\lim_{x \to \infty} \frac{x}{\sqrt{9x^2 - 4}}$.

(b) Calculate $\lim_{x \to \infty} \sqrt{x^2 + 9x} - x$.

(2) An airplane, flying east at 200 miles per hour, goes over a certain town at 11:00 A.M., and a second plane at the same altitude, flying north at 300 miles per hour, goes over the town at noon. How fast are the planes separating at 1:00 P.M.? Simplify your answer.

(3) Find the absolute maximum value and the absolute minimum value for

$$f(x) = x(x-6)^2 = x^3 - 12x^2 + 36x$$

on the interval $[1, 3]$.

Absolute Maximum Value: $\boxed{}$

Absolute Minimum Value: $\boxed{}$

(4) (a) Use that $f(x+h) \approx f(x) + hf'(x)$ to explain why $(27.27)^{1/3} \approx 3.01$. Fill in the boxes below (but this is not sufficient for an explanation).

$f(x) =$ $\boxed{}$

$x =$ $\boxed{}$

$h =$ $\boxed{}$

(b) Using the Mean Value Theorem, explain why $(27.27)^{1/3} \leq 3.01$.

(5) For this page and the next page, $f(x) = \dfrac{x}{x^2+3}$. You may use the following valuable information concerning $f(x)$:

$$f'(x) = -\frac{x^2-3}{(x^2+3)^2} \quad \text{and} \quad f''(x) = \frac{2x(x^2-9)}{(x^2+3)^3}.$$

(a) At what point does $f(x)$ cross the x-axis? (Give the x and y coordinates.)

(b) On what intervals is $f(x)$ increasing?

(c) On what intervals is $f(x)$ decreasing?

(d) What are the local maximum values of $f(x)$?

(e) What are the local minimum values of $f(x)$?

(f) On what intervals is $f(x)$ concave up?

(g) On what intervals is $f(x)$ concave down?

(h) What are the inflection points of $f(x)$? (Give the x and y coordinates.)

(i) There is a horizontal asymptote. Write down its equation.

(j) Draw the graph of $y = f(x)$.

(6) Find the points P and Q on the curve $y = x^2/4,\ 0 \leq x \leq 2\sqrt{5}$, which are closest to and furthest from the point $(0, 4)$.

$P =$ $\boxed{}$

$Q =$ $\boxed{}$