MATH 141: TEST 1

Name _____

Instructions and Point Values: Put your name in the space provided above. Work each problem below and show <u>ALL</u> of your work. You do not need to simplify your answers unless the instructions indicate to. Do <u>NOT</u> use a calculator.

Problem (1) is worth 18 points.   Problem (5) is worth 10 points.
Problem (2) is worth 24 points.   Problem (6) is worth 12 points.
Problem (3) is worth 10 points.   Problem (7) is worth 14 points.
Problem (4) is worth 12 points.

(1) Calculate each of the following limits, and put your answers in the appropriate boxes. Be sure to show work on each part. Make sure your work relates to material covered in this course.

(a) $\lim\limits_{x \to 2} \dfrac{x^2 - 4}{x^2 - x - 2} = \boxed{}$

(b) $\lim\limits_{x \to 0^-} \dfrac{x}{|x|} = \boxed{}$

(c) $\lim\limits_{x \to 0} \dfrac{\sin(3x)}{\sin(2x)} = \boxed{}$

(2) Calculate each of the derivatives below, and put your answers in the appropriate boxes. You do not need to show any work on this page.

(a) $\dfrac{d}{dx}(x\cos x) =$ [   ]

(b) if $y = \sqrt{x(x+2)}$, then $y' =$ [   ]

(c) if $f(t) = \dfrac{t+1}{\sin t}$, then $f'(t) =$ [   ]

(d) if $f(x) = \dfrac{1}{30x}$, then $f^{(5)}(1) =$ [   ]

(3) Find the equation of the tangent line to $y = x^6 - 2x^4 + 3x^3 - 1$ at the point $(1, 1)$.

Equation:

(4) (a) Express $f'(x)$ as a limit (i.e., what is the definition of $f'(x)$ assuming it exists).

(b) Let $f(x) = \sqrt{2x + 3}$. Using the definition of a limit (from part (a)), show that $f'(x) = 1/\sqrt{2x + 3}$.

(5) Calculate the slope of the tangent line to the graph of $2x^3 + 2xy + y^3 = 14$ at the point $(1, 2)$.

Slope:

(6) Let $f(x)$ and $g(x)$ be functions satisfying

$$f'(x) = 1 + \frac{x^2}{(x^3 + 2)^2}, \quad g'(x) = \frac{x}{\sqrt{x^2 + 3}}$$

$$g(1) = 0, \quad f(1) = 1, \quad \text{and} \quad f(0) = \frac{-1}{18}.$$

Let $H(x) = f(g(x))$. Using some of the information above (but not all of it), calculate $H'(1)$. (You must show appropriate work to receive credit.)

(7) Carolina Avenue and Calculus Highway intersect perpendicular to one another at a place known as Four Corners. A police officer is driving 40 m.p.h. toward Four Corners going North on Carolina Avenue. When she is 30 feet from Four Corners, she clocks another vehicle 40 feet past the intersection going West on Calculus Highway. She clocks the vehicle at 80 m.p.h. which means the distance BETWEEN THE TWO VEHICLES is increasing at a rate of 80 m.p.h. How fast is the second vehicle (the one not driven by the police officer) going? Simplify your answer.

(Comment: Being 30 feet from Four Corners means you are $30k$ miles from Four Corners for some constant k. If you want to convert all distances to miles, then simply use this constant k. You do not need to know its value. In the end, the answer will not depend on the value of k.)

ANSWER: | _____ m.p.h. |