MATH 141: TEST 1

Name ________________________________

Instructions and Point Values: Put your name in the space provided above. Work each problem below and show ALL of your work. You do not need to simplify your answers. Do NOT use a calculator.

Problem (1) is worth 9 points.
Problem (2) is worth 9 points.
Problem (3) is worth 16 points.
Problem (4) is worth 24 points.
Problem (5) is worth 12 points.
Problem (6) is worth 16 points.
Problem (7) is worth 14 points.

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(1) Evaluate $\lim\limits_{x\to 1} \dfrac{x^4 - x^3 - 2x + 2}{x^3 + x^2 - x - 1}$.

(2) Evaluate $\lim\limits_{x\to 0} \dfrac{\sin(2x)(\cos(x) - 1)}{x^2}$.

(3) Let

$$f(x) = \begin{cases} x^2 + 1 & \text{if } x \geq 1 \\ -x & \text{if } x < 1. \end{cases}$$

Answer each of the following. If a limit does not exist, simply write, "The limit does not exist." Otherwise determine its value. If you draw an appropriate graph of $y = f(x)$, you may answer part (a) and (b), but <u>NOT</u> (c) or (d), without showing any further work.

(a) Evaluate $\lim_{x \to 1^+} f(x)$.

(b) Evaluate $\lim_{x \to 1^-} f(x)$.

(c) Evaluate $\lim_{x \to 1} f(x)$. Explain.

(d) Is $f(x)$ continuous at 1? Explain.

(4) Calculate each of the derivatives below, and put your answers in the appropriate boxes. You do not need to show any work on this page.

(a) $\frac{d}{dt}\left(t^2+1\right)^{24} = \boxed{}$

(b) $\frac{d}{dx}\cos(\sqrt{x}) = \boxed{}$

(c) if $f(x) = (x+1)^{10}\sqrt{x}$, then $f'(x) = \boxed{}$

(d) $\frac{d}{dx}\left(\frac{x-2}{\sin(x)}\right) = \boxed{}$

(5) Find the equation of the tangent line to $y = x^3(x+1)^2$ at the point $(1,4)$.

(6) Let $y = \sin\left(\sqrt{\cos(x)}\right)$. Calculate y' by using the Chain Rule in the form

$$\frac{dy}{dx} = \frac{dy}{dv} \times \frac{dv}{du} \times \frac{du}{dx}.$$

You should use the boxes below to clarify your answer y' as well as the values of u, v, and y that you using to evaluate the expressions on the right-hand side above. You should not use $u = x$, $v = u$, or $y = v$.

$u = \boxed{}$

$v = \boxed{}$

$y = \boxed{}$

$y' = \boxed{}$

(7) A person experimentally observes that if he allows his head to contact a particular brick wall at a fast enough speed, his head will swell up at the rate of 1200 cubic inches per second. Before conducting any experiments his head was the shape of a perfect sphere of radius 4 inches. Assuming his head is still this shape when his head contacts a brick wall as above and that his head remains the shape of a sphere as it swells up, how fat is his head (that is, what is the radius of his head) when it's radius is increasing at the rate of 3 inch per second? (The volume of a sphere is $(4/3)\pi r^3$ where r represents the radius of the sphere.)

Answer: | inches |