MATH 141: FINAL EXAM

Name ______________________________

Instructions and Point Values: Put your name in the space provided above. Check that your test contains 15 different pages including one blank page. Work each problem below and show ALL of your work. Unless stated otherwise, you do not need to simplify your answers. You should NOT use any Calculus material in your answers that is unrelated to this course. Do NOT use a calculator.

There are 300 total points possible on this exam. There are 2 parts. The first part consists of 20 problems each worth 10 points. The second part consists of 5 problems each worth 20 points.

PART I. Calculate each of the following or answer the question that is asked. An asterisk ($*$) next to a problem number indicates that you do not need to show work for that problem. You should show work for every other problem.

(1) the equation of the tangent line to $y = x^2 - x$ at the pont $(1, 0)$

(2) $\displaystyle\lim_{x\to 0} \frac{\sin(3x)}{x}$

(3)* $\lim_{x \to 3^-} \frac{|x-3|}{x-3}$

(4) $\lim_{x \to \infty} \left(\sqrt{x^2+10x} - \sqrt{x^2-4x}\right)$

(5) $\lim_{h \to 0} \frac{\cos(\sqrt{x+h}) - \cos(\sqrt{x})}{h}$

(6)* $f'(t)$ if $f(t) = \cos\left(\sqrt{t}\right)$

(7)* y' if $y = \left(x+1\right)^{23}(x^3+1)^{17}$

(8)* $\dfrac{d}{dx}\left(\displaystyle\int_{x^2}^{2x} 5^t\,\sqrt{t^2+4}\;dt\right)$

(9) A ladder 10 feet long is leaning against a wall 14 feet tall that is perpendicular to the ground. When the tip of the ladder is 8 feet above the ground, the tip is falling at a rate of 12 feet per second. At that particular moment, how fast is the opposite end of the ladder (on the ground) moving away from the wall?

(10) Suppose that y satisfies the differential equation $y' = \dfrac{2x-1}{3y^2}$ and that $y = 2$ when $x = 1$. What is the value of y when $x = 8$? Simplify your answer.

(11) the area of the region bounded by the graphs of $y = x^2$ and $y = \sqrt{x}$

(12) the absolute *minimum* value of the function $f(x) = 2x^3 - 9x^2$ for x satisfying $-1 \leq x \leq 2$

(13) $\int \sqrt{x}(2x-1)(x+3)\,dx$

(14) $\displaystyle\int_1^{\sqrt{6}} \frac{2x}{\sqrt{x^2+3}}\,dx$

(15) $\displaystyle\int (2x^3+1)\sin\left(2x+x^4\right)dx$

(16) $\displaystyle\int \frac{(x-1)(3x+5)}{\sqrt{x+1}}\,dx$

(17) Jill's boat is sinking 4 miles from the nearest point P on a straight shoreline. Jill swims to shore and jogs to the nearest town 8 miles down the shoreline from P. If she swims at a rate of 3 miles per hour and jogs at a rate of 5 miles per hour, how many miles from P should she come to shore if she wishes to reach the town as quickly as possible? Justify that your answer minimizes the time traveled.

(18) the derivative of $\dfrac{2}{x+1}$ by using the definition of the derivative, namely

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

(Comment: To receive credit for the problem make sure you use only the definition of the derivative and information about limits.)

(19) $\int_0^2 \sqrt{4-x^2}\,dx$ (Hint: Do not try to calculate the value of the integral directly but instead begin with a graph of an area that the integral represents.)

(20) the equation of the tangent line to the graph of

$$2\sin\left(\frac{\pi x y}{2}\right) = y^2 + x$$

at the point $(1,1)$

PART II. Answer each of the following. Make sure your work is clear. If you do not know how to answer a problem, tell me what you know that you think is relevant to the problem. If you end up with an answer that you think is incorrect, tell me this as well. Better yet, tell me why you think it is incorrect. In other words, let me know what you know.

(1) Calculate the integral $\int_a^b f(x)\,dx$ boxed below in the following way. Divide the interval $[a,b]$ into n equal subintervals, calculate the area of the corresponding circumscribed polygon, and then let $n \to \infty$. You should make use of the formula

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}.$$

Your final answer should be a number.

$$\boxed{\int_2^6 (2x-3)\,dx}$$

(2) For this page and the next page, $f(x) = \dfrac{x^2+1}{(x-1)^2}$. The following information is given to you (you may use it):

$$f'(x) = \frac{-2(x+1)}{(x-1)^3} \quad \text{and} \quad f''(x) = \frac{4(x+2)}{(x-1)^4}.$$

(a) Find all the critical points for $y = f(x)$.

(b) Find all local maximum values of $f(x)$.

(c) Find all local minimum values of $f(x)$.

(d) Find all asymptotes for the graph of $y = f(x)$.

(e) On what intervals is the graph of $y = f(x)$ concave up?

(f) Find all the inflection points for $y = f(x)$.

(g) Graph $y = f(x)$.

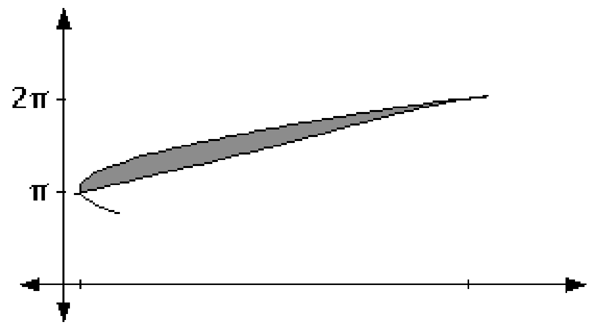
(3) Explain why the point $(2, 1)$ is the point on the graph of $y = 16x^{-4}$, with $x > 0$, that is nearest the origin.

(4) The graphs of

$$x = \frac{2^y - \cos\left(y + \frac{3\pi}{4}\right)}{y} \quad \text{(on the left)}$$

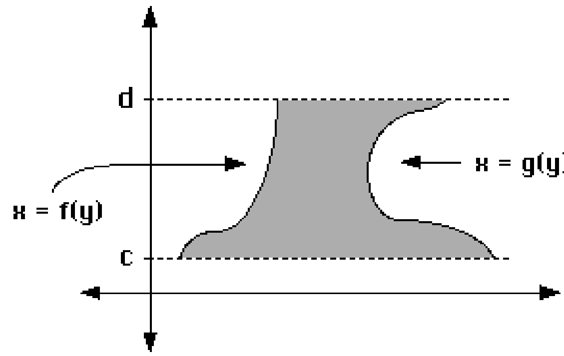
and

$$x = \frac{2^y + \sin\left(y + \frac{3\pi}{4}\right)}{y} \quad \text{(on the right)}$$



intersect when $y = \pi$ and $y = 2\pi$ as shown. The region between these curves for $\pi \leq y \leq 2\pi$ is revolved about the x-axis to form a solid. What is the volume of this solid?

(5) Given the graphs of $x = f(y)$ and $x = g(y)$ to the right, express each of the following as an integral.



(a) the area of the shaded region

(b) the volume of the solid obtained by revolving the shaded region about the y-axis

(c) the volume of the solid obtained by revolving the shaded region about the line $y = d$