TEAM PROBLEMS FEBRUARY, 2000

Instructions: Answer as many of the problems below as you can. At the end of the time allotted, turn in a list of your answers. Your answers should be expressed in simplest form. Exact answers are required on all problems (not numerical approximations).

1. Simplify $\sqrt{17+\sqrt{273}}-\sqrt{17-\sqrt{273}}$.

2. What is the value of the coefficient of $x^{2000999}$ in the expansion of

$$(x+1)(x-2)^2(x+3)^3(x-4)^4\cdots(x+1999)^{1999}(x-2000)^{2000} \ ?$$

3. Let A, B, and C be points such that $AB = 9$, $AC = 8$, and $\angle ABC = 60°$. There are two different possibilities for the value of BC. Suppose ℓ is the largest of these and s is the smallest. Then what is the value of $\ell - s$?

4. If a thin 10 foot ladder leans against a wall, a right triangle is formed with the hypotenuse being the ladder and the legs being formed by the ground and the wall. If the highest point of the ladder is taken at random between 0 and 10 feet, what is the probability that the area of the right triangle formed is greater than 15?

5. Let P denote the point $(4,2)$, and consider the circle $x^2+y^2=2$. A line which passes through P and intersects the circle in exactly one point is said to be *tangent* to the circle. There are two such tangent lines. At what two points do they intersect the circle?

6. If 95^{2000} is written in base 2, then what is the sum of its 10 right-most bits (binary digits)?

7. Observe that $649^2 - 13 \times 180^2 = 1$. There is one pair $(a,b) \neq (649,180)$ with each of a and b a positive integer $< 10^8$ and with $a^2 - 13b^2 = 1$. Determine (a,b).

8. The polynomial $x^n + 2x^2 + 3x - 4$ has exactly one positive real root. Denote it by α_n. As n gets larger and larger (as n approaches infinity), the value of α_n gets closer and closer to some real number β (α_n approaches β) in the sense that the value of $|\alpha_n - \beta|$ can be made arbitrarily small by taking n to be sufficiently large. What is the value of β?

Solutions are located at the website `http://www.math.sc.edu/~filaseta/contests/contests.html`