TEAM PROBLEMS* February, 1999

(Calculators are permitted on this competition.)

T1. A fair coin is to be flipped at random 6 times. What is the probability that the coin will land on heads face-up more often than on tails face-up?

T2. If n and j are integers with $0 \leq j \leq n$, then the binomial coefficient $\binom{n}{j}$ (which is read "n choose j") is defined by $\binom{n}{j} = \frac{n!}{j!(n-j)!}$. What is the value of

$$\sum_{j=0}^{16} 2^j \binom{16}{j}\,?$$

T3. Two points A and B are on a circle of radius one. The distance from A to B is one. If C is a point on the circle such that the center of the circle is inside ΔABC, then what is the measure of $\angle C$?

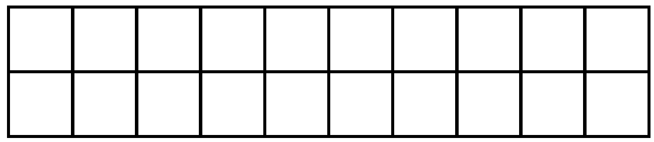
T4. In the decimal expansion of $\frac{1}{1999}$, what are the first four digits (in order) after the first 100 digits after the decimal? In other words, if $\frac{1}{1999} = 0.d_1d_2d_3\dots$ where the d_j denote digits, then what is $d_{101}d_{102}d_{103}d_{104}$?

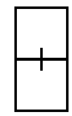
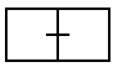
T5. The graphs of $y = 4x^3 - \pi x^2 + 3x - 1$ and $y = 8x^2 - 5$ intersect in exactly three points. If these points are (x_1, y_1), (x_2, y_2), and (x_3, y_3), then what is the value of $x_1 + x_2 + x_3$?

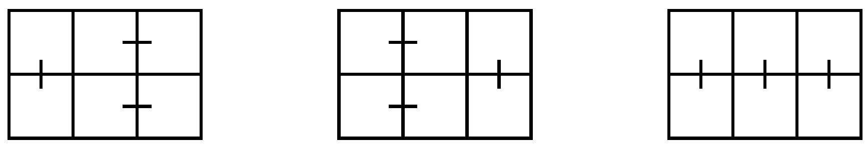
T6. Let $f(x) = \cos x + 3\sin x$. Suppose t is a real number such that $f(t) \geq f(x)$ for all real x. Calculate the exact value of $\sin t$.

*For the solutions, see `http://www.math.sc.edu/~filaseta/contests.html`.

T7. A 2×10 board



is to be completely covered by 10 dominoes (i.e., 2×1 or 1×2 boards shaped  or , respectively). In how many ways can this be done? For example, there are 3 different ways to cover a 2×3 board with dominoes illustrated below.



T8. The polynomial $x^2 - 4x + 5$ can be multiplied by a non-zero polynomial $g(x)$ (having integer coefficients) with the resulting product having non-negative integer coefficients (and, hence, degree at least two). What is the smallest possible degree for such a $g(x)$?