Math 788T Study Guide

1. Distribution of integers that are sum of two squares of integers. The Greedy Algorithm.

2. Farey sequences; definition and main properties; the number of Farey fractions in an interval.

3. Estimate for the number of lattice points on a curve, by using Farey fractions.

4. Divided differences. Definition via interpolating polynomials, main properties, recurrence and other formulas involving divided differences.

5. Distribution of squarefree numbers - general information.

6. Distribution of squarefree numbers in short intervals. Formulation of the problem and its reduction to estimating the number of integer points close to a smooth curve.

7. Estimates, based on divided differences (first, second and third). Corollaries implied by these estimates.

8. Polynomial identities and their use to obtain information about the spacing of the lattice points close to a curve.

9. Auxiliary results - estimates of sums via integrals.

10. Squarefull numbers - general information.

11. Distribution of squarefull numbers in short intervals. Formulation of the problem and its reduction to the problem of estimating the number of lattice points close to a curve.

12. The geometry of the set of integer points close to a curve. Convexity conditions.

13. Estimates for the size of a set of integer points close to a curve, assuming convexity of the set.

14. Auxiliary results - estimating

$$\min_{0<x\leq H} A_1x^{a_1} + A_2x^{a_2} + \cdots + A_nx^{a_n} + Bx^{-b}.$$

15. The method of Swinnerton-Dyer - general idea, prerequisites, initial identities, outline of the proof.

16. Auxiliary results - estimates of

$$\sum_{n\leq x, n\equiv 0 \,(\mathrm{mod}\, d)} n^{\alpha}.$$