PAlmetto Number Theory Series

University of South Carolina, December 1-2, 2012

Invited Talks

Alyson Deines (University of Washington)

On an algorithm for computing degrees of parametrizations of elliptic curves by Shimura curves

Let E be an elliptic curve over $\mathbb{Q}$ of conductor N. Then E has a modular parameterization, specifically there is a surjective morphism ϕ from the modular curve $X_0(N)$ to E. The degree of this map, m_E, is called the modular degree. There are many theorems and conjectures relating the modular degree m_E of an elliptic curve to the modular form f_E associated to E, of particular interest is the relation to congruence primes. Unfortunately, generalizing to number fields, we no longer always have modular curves. Takahashi and Ribet use the Jaquet-Langlands correspondence to parameterize elliptic curves over $\mathbb{Q}$ by Shimura curves. I will examine how this generalizes to modular elliptic curves and in general modular abelian varieties over number fields.

Benedict Gross (Harvard University)

On the values of zeta functions at negative integers and the dimension of spaces of modular forms

Although the zeta function is named after Riemann, it was Euler who discovered most of its amazing properties. I will review his results on the rationality and integrality of its values at negative integers. I will then show how these values occur in the calculation of the volume of some adelic coset spaces and give information on dimension of spaces of modular forms.

Wei Ho (Columbia University/Princeton University)

Ranks of elliptic curves over quadratic extensions

We will recall how parametrizations of Selmer elements for elliptic curves by orbits of representations of algebraic groups, combined with counting techniques coming from the geometry of numbers, allow us to compute the average size of Selmer groups for elliptic curves in various families. We will discuss an example for which such an average gives a new application to ranks of elliptic curves over quadratic fields.

This is joint work with Manjul Bhargava.

(No previous knowledge of Selmer groups is needed!)

Winfried Kohnen (Ruprecht-Karls-Universität Heidelberg)

Sign changes of Fourier coefficients of cusp forms

This will be a survey talk about recent results about sign changes of Fourier coefficients/eigenvalues of cusp forms, both in the case of elliptic modular forms as well as in the case of Siegel modular forms of degree two.

Carl Pomerance (Dartmouth College)

Balanced subgroups of the multiplicative group

Say a subgroup G of the multiplicative group modulo n is *balanced* if with the usual set of representatives, each coset of G has the same number of elements in the interval $(0, n/2)$ as in $(n/2, n)$. (Here, $n > 2$.) It was recently shown that this concept is related to the ranks of certain elliptic curves over function fields. Here we give a useful criterion for a subgroup to be balanced and we study the problem from a statistical point of view. For example, given a fixed integer k, how common is it for an integer n coprime to k to have the property that $\langle k \bmod n \rangle$ is a balanced subgroup of the multiplicative group modulo n? This talk is based on joint work with Douglas Ulmer.

Matt Young (Texas A&M University)

Restrictions of automorphic forms and L-functions

Abstract: I will discuss recent progress on some questions about automorphic forms having the common theme that one begins with an automorphic form on some space, and then one wishes to understand its behavior when restricted to a natural subspace. Does the form vanish along this subspace? How large can it be? These types of questions have connections to mathematical physics (quantum chaos), representation theory, and geometry. I will talk about some particularly nice examples where these investigations lead naturally to families of L-functions, where tools from analytic number theory play a crucial role. These are joint works with Valentin Blomer, Rizwan Khan, Xiaoqing Li, Sheng-Chi Liu, and Riad Masri, in various combinations.

Contributed Talks

Saikat Biswas (Georgia Tech)

Tamagawa Torsors of an Abelian Variety

For an abelian variety A defined over a number field K, we define the Tamagawa torsors of A at a prime v of K to be the set of A-torsors defined over the completion K_v of K at v that are split by an unramified extension. It turns out the set of such torsors is finite, having cardinality equal to the Tamagawa number of A at v (which also prosaically explains the terminology). In this talk, we will discuss some arithmetic properties of the Tamagawa torsors including its relation to the Selmer group as well as the Shafarevich-Tate group of A. Finally, following Mazur's theory of visibility, we will give conditions under which non-trivial Tamagawa torsors may be 'visualized' as Mordell-Weil points on another variety B also defined over K. We also show how our visibility theorem provides theoretical evidence for the Birch and Swinnerton-Dyer Conjecture.

Nathan Green (Brigham Young)

Integrality preserving lifts for level 3 forms

In his 1995 paper, Zagier described families of lifts of modular forms of weight 0 in level 1 to modular forms of weight 1/2 of level 4. It has been shown that these lifts generalize to spaces of weakly modular forms of all negative weight and all levels. We show that for level 3, the lifts preserves integrality of Fourier Coefficients.

Michael Griffin (Emory University)

SU(2)-Donaldson invariants of the complex projective plane.

There are two families of Donaldson invariants for the complex projective plane, corresponding to the SU(2)-gauge theoryand the SO(3)-gauge theory with non-trivial Stiefel-Whitney class. In 1997 Moore and Witten conjectured that theregularized u-plane integral on CP2 gives the generating functions for these Donaldson invariants. In earlier work, Malmendier and Onoproved the conjecture for the SO(3)-gauge theory. We complete the proof of the conjecture byconfirming the claim for the SU(2)-gauge theory.

Marie Jameson (Emory University)

Quadratic polynomials, period polynomials, and Hecke operators

For every positive non-square $D \equiv 0, 1 \pmod 4$ and every positive even integer k, Zagier defined a beautiful function by

$$F_k(D; x) := \sum_{\substack{a,b,c \in \mathbb{Z}, a<0, \\ b^2-4ac=D}} \max(0, (ax^2 + bx + c)^{k-1}).$$

Here we use the theory of periods to give identities and congruences which relate various values of $F_k(D; x)$.

Rodney Keaton (Clemson University)

A congruence between Ikeda lifts and non-Ikeda lifts

In 1976, Ribet completed the proof of what is now called the Herbrand-Ribet Theorem. A crucial piece of his proof involved the use of a congruence between certain Eisenstein series and cuspidal eigenforms. In the setting of Siegel modular forms, the Ikeda lift provides us with an analogue to the Eisenstein series. Recently, Katsurada has conjectured under precisely which conditions a congruence between Ikeda lifts and non-Ikeda lifts should occur. In this talk we will present a result which provides evidence for this conjecture.

Robert Lemke Oliver (Emory University)

Representation by ternary quadratic forms

The problem of determining when an integral quadratic form represents every positive integer has received much attention in recent years, culminating in the 15 and 290 Theorems of Bhargava-Conway-Schneeberger and Bhargava-Hanke. If a form fails to represent all integers, perhaps the next best thing would be if it were to represent all locally-represented integers. Indeed, such forms exist and are called regular, and Jagy, Kaplansky, and Schiemann proved that there are at most 913 such that are ternary; however, only 899 of these are actually known to be regular. We consider the remaining 14 forms, and establish the regularity of each under the generalized Riemann Hypothesis, following the method pioneered by Ono and Soundararajan. Moreover, we consider the exceptional arithmetic consequences if a large, locally represented integer is not globally represented by a ternary quadratic form, proving that some Dirichlet L-function would necessarily have a Siegel zero or that some quadratic twist of an elliptic curve would have an unusally large Tate-Shafarevich group.

Adele Lopez (Emory University)

Prescribing imaginary quadratic fields with 2-class group of type $(2, 2^\ell)$

The abstract is: For any given positive integer ℓ, we prove there are infinitely many imaginary quadratic fields with 2-class group of type $(2, 2^\ell)$. We do this by using the circle method to prove that certain polynomials take on infinitely many values that are the sum of two primes, where each prime is in a specific congruence class.

Dermot McCarthy (Texas A&M University

Transformation properties of finite field hypergeometric functions

An important feature of classical hypergeometric series is their powerful transformation and summation formulas. In this talk we will introduce finite field hypergeometric functions, which are analogous to the classical series. We will outline recent work on developing transformation properties for these functions, which mirror those of the classical case. We will also discuss how these new transformations have contributed to relating special values of the function to Fourier coefficients of a certain Siegel modular form.

Dong Quan (University of British Columbia

Generalized Mordell curves, generalized Fermat curves, and the Hasse principle

I will show that for each prime p congruent to 1 (mod 8), there exists a threefold X_p in $\mathbb{P}_6$ such that the existence of certain rational points on X_p produces families of generalized Mordell curves and of generalized Fermat curves that are counterexamples to the Hasse principle explained by the Brauer-Manin obstruction. Furthermore, in this talk, I will also introduce a notion of the descending chain condition(DCC) for sequences of curves, and prove that there are sequences of generalized Mordell curves and of generalized Fermat curves satisfying DCC.

Lola Thompson (University of Georgia)

Sums of distinct divisors

Following Srinivasan, an integer $n \geq 1$ is called *practical* if every natural number in $[1, n]$ can be written as a sum of distinct divisors of n. This motivates us to define $f(n)$ as the largest integer with the property that all of $1, 2, 3, \ldots, f(n)$ can be written as a sum of distinct divisors of n. (Thus, n is practical precisely when $f(n) \geq n$.) We think of $f(n)$ as measuring the "practicality" of n; large values of f correspond to numbers n which we term *practical pretenders*.

In this talk, we will describe the distribution of the practical pretenders, as well as the maximal order of f when restricted to non-practical inputs; these results improve upon theorems of Saias and Hausman and Shapiro in the literature on practical numbers. This talk is based on joint work with Paul Pollack.

Hua Wang (Georgia Southern University)

Compositions, tilings, and numerical strings

We consider colored-compositions of an integer, a generalization of traditional integer compositions. Through tiling representations, we show bijections between various types of colored-compositions and numeric strings. One of the approaches used generalizes the classic bijection between traditional compositions and binary strings. This is joint work with Charles Dedrickson.

John Webb (Wake Forest University)

Bounds for p-core partitions by calculating cusp constants

Let p be prime. We say a partition of n is p-core if none of the hook lengths in the corresponding Ferrers-Young diagram for the partition is divisible by p. For $p \geq 29$, we obtain optimal bounds for the number of p-core partitions by closely approximating the cusp constant of an associated eta-quotient. This is joint work with Jeremy Rouse and builds upon previous work of Granville and Ono, Kim and Rouse, as well as others.

Hang Xue (Columbia University)

The height of a canonical point in the Jacobian of a genus four curve

In this talk, we construct a quadratic point in the Jacobian of a non-hyperelliptic curve of genus four over a global field. We then compute the Neron–Tate height of this point in terms of the self-intersection of the (admissible) dualizing sheaf and some canonically defined local invariants. We show that the height of this point satisfies the Northcott property. When the reduction of the curve is simple, we compute explicitly the local invariants.

Yongqiang Zhao (University of Wisconsin)

Counting cubic extensions of rational function fields

In this talk, I will explain how to use some elementary Algebraic Geometry to count cubic extensions of rational function fields with bounded discriminant. For each cubic field, we assign an invariant, the so called Maroni Invariant. This invariant gives a natural explanation why the counting funtion of cubic fields has a secondary term.
