PAlmetto Number Theory Series

Abstracts for December 6 - 7, 2014 Meeting

Plenary Speakers

James Maynard (Oxford University).

Title: *Large and small gaps between primes*

Abstract: We will discuss a modification of the Selberg sieve, and how this can be used to show the existence of unusually small and unusually large gaps between primes.

Lillian Pierce (Duke University).

Title: *Class numbers of quadratic number fields: a few highlights on the timeline from Gauss to today*

Abstract: Each number field (finite field extension of the rational numbers) has an invariant associated to it called the class number (the cardinality of the class group of the field). Class numbers pop up throughout number theory, and over the last two hundred years people have been considering questions about the growth and divisibility properties of class numbers. We will focus on class numbers of quadratic extensions of the rational numbers, surveying some key results in the two centuries since the pioneering work of Gauss, and then turning to very recent joint work of the speaker with Roger Heath-Brown on averages and moments associated to class numbers of imaginary quadratic fields.

Anthony Várilly-Alvarado (Rice University).

Title: *Rational points on K3 surfaces.*

Abstract: K_3 surfaces form a class of algebraic surfaces that are arithmetically mysterious. We don't even have a solid conjectural understanding for the kinds of invariants that might control the existence of rational points on them. A first stumbling block in this direction is our incomplete grasp of their Brauer groups (over number fields!). My plan in this talk is to convey why K_3 surfaces and their Brauer groups have become a focus of research in recent years. I will describe how the Brauer group of a K_3 surface can affect the existence and distribution of rational points, and I will sketch an idea for how to try to understand what these Brauer groups look like in general.

Invited Postdoctoral Speaker

Sun Kim (University of Illinois at Urbana-Champaign)

Title: *Sums of divisors functions and Bessel function series*

Abstract: On page 335 in his Lost Notebook, Ramanujan recorded without proofs two identities involving finite trigonometric sums and doubly infinite series of Bessel functions. These two identities are intimately connected with the classical circle and divisor problems, respectively. There are three possible interpretations for the double series of these identities. The first identity has been proved under all three interpretations, and the second under two of them. Furthermore, several analogues of them were established, and they were extended to Riesz sum identities as well. In this talk, we review these results, and also provide analogous identities for the weighted sums of divisors functions. In particular, two of them yield a generalization of the Riesz sum identity for $r_6(n)$.

Invited Graduate Student Speaker

Chao Li (Harvard University).

Title: *Level raising mod 2 and arbitrary 2-Selmer ranks*

Abstract: We prove a level raising mod $p = 2$ theorem for elliptic curves over $\mathbb{Q}$, generalizing theorems of Ribet and Diamond-Taylor. As an application, we show that the 2-Selmer rank can be arbitrary in level raising families. We will begin by explaining our motivation from W. Zhang's approach to the p-part of the BSD conjecture. Explicit examples will be given to illustrate different phenomena compared to odd p. This is joint work with Bao V. Le Hung.

Contributed Talks

Michael Bush (Washington and Lee University).

Title: *Non-abelian generalizations of the Cohen-Lenstra heuristics*

Abstract: In the context of quadratic fields, the Cohen-Lenstra Heuristics make precise conjectures about how often one should expect a finite abelian group to appear as the odd part of the class group as one considers fields ordered by discriminant. Over the last several years, Nigel Boston, Farshid Hajir and I have formulated analogous non-abelian heuristics in which the p-class group (p an odd prime) is replaced with the Galois group of the maximal unramified p-extension of the field. I'll discuss some recent developments in both the formulation of our conjectures and the collection of numerical evidence.

Josh Harrington (Cedar Crest College).

Title: *Two questions concerning covering systems*

Abstract: Ever since Erdos introduced the concept of a covering system in 1950, many questions have arisen regarding the existence of certain types of covering systems. Two of the most famous questions regarding covering systems are the odd covering problem and the minimum modulus problem. In this talk we ask two questions that are related to these famous questions and provide results toward each.

Jesse Kass (University of South Carolina).

Title: *How to count two points in the plane?*

Abstract: What is the asymptotic distribution of pairs of rational points in the plane? More formally, for a given height function, what is the asymptotic behavior of the function counting rational points of bounded height on the Hilbert scheme of 2 points in the projective plane? In my talk I will discuss joint work with Frank Thorne on this problem.

Alicia Lamarche (Shippensburg University).

Title: *Generating composite sequences of the form $k(2^n + F_n) + 1$*

Abstract: In 1956, Riesel showed that there exist infinitely many positive integers k such that $k2^n - 1$ is composite for all integers $n > 0$, and in 1960 Sierpinski proved a similar theorem concerning sequences of the form $k2^n + 1$. In 2012, the second author showed that there exist infinitely many k such that the sequence $F_n + k$ is composite for all $n > 0$, where F_n denotes the nth term of the Fibonacci sequence. We consider a combination of these ideas, and establish results concerning the existence of k such that the sequence $k(2^n + F_n) \pm 1$ is composite for all $n > 0$. This is joint work with Kellie Bresz, Lenny Jones, and Maria Markovich.

Frank Patane (University of Florida).

Title: *An identity connecting theta series associated with binary quadratic forms of discriminant Δ and Δp^2*

Abstract: I will state and prove a new identity which connects theta series associated with binary quadratic forms of idoneal discriminants Δ and Δp^2, for p a prime. I then illustrate how to use this identity to derive Lambert series identities and hence product representation formulas for certain forms. Last but not least, I discuss generalizations to non-idoneal discriminants and the resulting theta series identities one may derive.

Drew Sills (Georgia Southern University).

Title: *A formula for the partition function that "counts"*

Abstract: An asymptotic formula for $p(n)$, precise enough to give the exact value, was given by Hardy and Ramanujan in 1918. Twenty years later, Hans Rademacher improved the Hardy-Ramanujan formula to give an infinite series that converges to $p(n)$. The Hardy-Ramanujan-Rademacher series is revered as one of the truly great accomplishments in the field of analytic number theory.

In 2011, Ken Ono and Jan Bruinier surprised the world by announcing a new formula which attains $p(n)$ by summing a finite number of complex numbers which arise in connection with the multiset of algebraic numbers that are the union of Galois orbits for the discriminant $-24n + 1$ ring class field.

Thus despite the fact that $p(n)$ is a combinatorial function, the known formulas for $p(n)$ involve deep mathematics, and are by no means "combinatorial" in the sense that they involve summing a finite or infinite number of complex numbers to obtain the correct positive integer value.

In this talk, I will present a combinatorial multisum expression for $D(n, k)$, the number of partitions of n with Durfee square of order k. Of course, summing $D(n, k)$ over $1 \le k \le \sqrt{n}$ yields $p(n)$. This, in turn leads to a natural approximation to p(n) as a polynomial with rational coefficients. Numerical evidence suggests that this polynomial approximation obtains accuracy comparable to that of the initial term of the Hardy-Ramanujan-Rademacher series.

The idea behind the formula is due to Yuriy Choliy, and the work was completed in collaboration with him.

Kate Thompson (Davidson College).

Title: *Local densities and quadratic forms*

Abstract: To study a positive definite integral quadratic form Q analytically, one examines its theta series Θ_Q–a modular form with weight, level and character determined by Q. The theory of local densities, developed by Siegel in the 1930s, is then used to understand the Fourier coefficients of Θ_Q. This technique is very powerful and is a crucial ingredient for major modern results on quadratic forms including the Bhargava-Hanke 290-Theorem (2005). This talk will discuss the way one generally applies this tool, as well as mention more recent

applications to quadratic forms defined over totally real number fields and Hilbert modular forms.

Lee Troupe (University of Georgia).

Title: *Bounded gaps between primes in $\mathbb{F}_q[t]$ with a given primitive root*

Abstract: A famous conjecture of Artin states that there are infinitely many prime numbers for which a fixed integer g is a primitive root, provided g is not -1 and g is not a perfect square. Thanks to work of Hooley, we know that this conjecture is true, conditional on the truth of the Generalized Riemann Hypothesis. Using a combination of Hooley's analysis and the techniques of Maynard-Tao used to prove the existence of bounded gaps between primes, Pollack has shown that (conditional on GRH) there are bounded gaps between primes with a prescribed primitive root. In this talk, we discuss the analogue of Pollack's work in the function field case; namely, that given a monic polynomial $g(t)$ which is not an ℓth power for any ℓ dividing $q-1$, there are bounded gaps between monic irreducible polynomials $P(t)$ in $\mathbb{F}_q[t]$ for which $g(t)$ is a primitive root (which is to say that $g(t)$ generates the group of units modulo $P(t)$). In particular, we obtain bounded gaps between primitive polynomials, corresponding to the choice $g(t) = t$.

Ali Uncu (University of Florida).

Title: *A new companion to Caparelli's partition theorem*

Abstract: In this talk we will revisit an identity (Capparelli's conjecture) proven by G. E. Andrews in 1994. Caparelli's conjecture states that the number of partitions of n into distinct parts not congruent to 1 or 5 mod 6 is equal to the number of partitions of n into distinct parts greater than 1 with minimal difference 2, where the difference is greater or equal than 4 unless consecutive parts add up to a multiple of 6 or are both multiples of 3.

We wil prove the following companion: The number of partitions of n into distinct parts not congruent to 1 or 5 mod 6 is equal to the number of partitions of n into distinct parts where odd indexed parts are not 1 mod 3 and even indexed parts are not 2 mod 3 and pairs $(2 + 3m, 1 + 3m)$ are not allowed in partition.

Hua Wang (Georgia Southern University).

Title: *Packing (colored) patterns in permutations*

Abstract: It is known that the maximum number of layered patterns is achieved in layered permutations. This general result allows the presentation of such optimal permutations with regard to all patterns of length three and most patterns of length four. When the pattern and permutations are colored, the problem, although still of the same nature, becomes different. We present background information and some preliminary studies on this topic.