Consecutive primes which are widely digitally delicate

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Dedicated to the fond memory of Ronald Graham

Abstract

We show that for every positive integer k, there exist k consecutive primes having the property that if any digit of any one of the primes, including any of the infinitely many leading zero digits, is changed, then that prime becomes composite.

1. Introduction

In 1978, M. S. Klamkin [19] posed the following problem.

> *Does there exist any prime number such that if any digit (in base* 10*) is changed to any other digit, the resulting number is always composite?*

In addition to computations establishing the existence of such a prime, the published solutions in 1979 to this problem included a proof by P. Erdős [6] that there exist infinitely many such primes. Borrowing the terminology from J. Hopper and P. Pollack [16], we call such primes *digitally delicate*. The first digitally delicate prime is 294001. Thus, 294001 is a prime and, for every $d \in \{0, 1, \ldots, 9\}$, each of the numbers

$$d94001, \quad 2d4001, \quad 29d001, \quad 294d01, \quad 2940d1, \quad 29400d$$

is either equal to 294001 or composite. The proof provided by Erdős consisted of creating a partial covering system of the integers (defined in the next section) followed by a sieve argument. In 2011, T. Tao [29] showed by refining the sieve argument of Erdős that a positive proportion (in terms of asymptotic density) of the primes are digitally delicate. In 2013, S. Konyagin [21] pointed out that a similar approach implies that a positive proportion of composite numbers n, coprime to 10, satisfy the property that if any digit in the base 10 representation of n is changed, then the resulting number remains composite. For example, the number $n = 212159$ satisfies this property. Thus, every number in the set

$$\{d12159, 2d2159, 21d159, 212d59, 2121d9, 21215d : d \in \{0, 1, 2, \ldots, 9\}\}$$

is composite. Later, in 2016, J. Hopper and P. Pollack [16] resolved a question of Tao's on digitally delicate primes allowing for an arbitrary but fixed number of digit changes to the beginning and end of the prime. All of these results and their proofs hold for numbers written in an arbitrary base b rather than base 10, though the proof provided by Erdős [6] only addresses the argument in base 10.

In 2020, the first author and J. Southwick [11] showed that a positive proportion of primes p, are *widely digitally delicate*, which they define as having the property that if any digit of p, *including any one of the infinitely many leading zeros of* p, is replaced by any other digit, then the resulting number is composite. The proof was specific to base 10, though they elaborate on other bases for which the analogous argument produces a similar result, including for example base 31; however, it is not even clear whether widely digitally delicate primes exist in every base. Observe that the first digitally delicate prime, 294001, is not widely digitally delicate since 10294001 is prime. It is of some interest to note that even though a positive proportion of the primes are widely digitally delicate, no specific examples of widely digitally delicate primes are known. Later in 2020, the authors with J. Southwick [9] gave a related argument showing that there are infinitely many (not necessarily a positive proportion) of composite numbers n in base 10 such that when any digit is inserted in the decimal expansion of n, including between two of the infinitely many leading zeros of n and to the right of the units digit of n, the number n remains composite (see also [10]).

In this paper, we show the following.

Theorem 1. *For every positive integer k, there exist k consecutive primes all of which are widely digitally delicate.*

Let $\mathcal{P}$ be a set of primes. It is not difficult to see that if $\mathcal{P}$ has an asymptotic density of 1 in the set of primes, then there exist k consecutive primes in $\mathcal{P}$ for each $k \in \mathbb{Z}^+$. On the other hand, for every $\varepsilon \in (0, 1)$, there exists $\mathcal{P}$ having asymptotic density $1-\varepsilon$ in the set of primes such that there do not exist k consecutive primes in $\mathcal{P}$ for k sufficiently large (more precisely, for $k \geq 1/\varepsilon$). Thus, the prior results stated above are not sufficient to establish Theorem 1. The main difficulty in using the prior methods to obtain Theorem 1 is in the application of sieve techniques in the prior work. We want to bypass the use of sieve techniques and instead give complete covering systems to show that there is an arithmetic progression containing infinitely many primes such that every prime in the arithmetic progression is a widely digitally delicate prime. This then gives an alternative proof of the result in [11]. After that, the main driving force behind the proof of Theorem 1, work of D. Shiu [26], can be applied. D. Shiu [26] showed that in any arithmetic progression containing infinitely many primes (that is, $an + b$ with $\gcd(a, b) = 1$ and $a > 0$) there are arbitrarily long sequences of consecutive primes. Thus, once we establish through covering systems that such an arithmetic progression exists where every prime in the arithmetic progression is widely digitally delicate, D. Shiu's result immediately applies to finish the proof of Theorem 1.

Our main focus in this paper is on the proof of Theorem 1. However, in part, this paper is to emphasize that the remarkable work of Shiu [26] provides for a nice application to a number of results established via covering systems. One can also take these applications further by looking at the strengthening of Shiu's work by J. Maynard [23]. To illustrate the application of Shiu's work in other context, we give some further examples before closing this introduction.

A Riesel number is a positive odd integer k with the property that $k \cdot 2^n - 1$ is composite for all positive integers n. A Sierpiński number is a positive odd integer k with the property that $k \cdot 2^n + 1$ is composite for all nonnegative integers n. The existence of such k were established in [25] and [27], respectively, though the former is a rather direct consequence of P. Erdős's work in [5] and the latter is a somewhat less direct application of this same work, an observation made by A. Schinzel (cf. [8]). A Brier number is a number k which is simultaneously Riesel and Sierpiński, named after Eric Brier who first considered them (cf. [8]). The smallest known Brier number, discovered by Christophe Clavier in 2014 (see [28]) is

$$3316923598096294713661.$$

As is common with all these numbers, examples typically come from covering systems giving an arithmetic progression of examples. In particular, Clavier established that any integer in the arithmetic progression

$$3770214739596601257962594704110\, n + 3316923598096294713661$$

is a Brier number. As 3770214739596601257962594704110 and 3316923598096294713661 are coprime, Shiu's theorem gives the following.

Theorem 2. *For every positive integer k, there exist k consecutive primes all of which are Brier numbers.*

Observe that as an immediate consequence the same result holds if Brier numbers are replaced by Riesel or Sierpiński numbers.

As another less obvious result to apply Shiu's theorem to, we recall a result of R. Graham [12] from 1964. He showed that there exist relatively prime positive integers a and b such that the recursive Fibonacci-like sequence

$$u_0 = a, \quad u_1 = b, \quad \text{and} \quad u_{n+1} = u_n + u_{n-1} \quad \text{for integers } n \geq 1, \tag{1}$$

consists entirely of composite numbers. The known values for admissible a and b have decreased over the years through the work of others including D. Knuth [20], J. W. Nicol [24], and M. Vsemirnov [30], the latter giving the smallest known such a and b (but notably the same number of digits as the a and b in [24]). The result has also been generalized to other recursions; see A Dubickas, A. Novikas and J. Šiurys [4], D. Ismailescu, A. Ko, C. Lee and J. Y. Park [17], and I. Lunev [22]. As the Graham result concludes with all u_i being composite, the initial elements of the sequence, a and b, are composite. However, there is still a sense in which one can apply Shiu's result. To be precise, the smallest known example given by Vsemirnov is done by taking

$$a = 106276436867 \quad \text{and} \quad b = 35256392432.$$

With u_j defined as above, one can check that each u_j is divisible by a prime from the set

$$\mathcal{P} = \{2, 3, 5, 7, 11, 17, 19, 23, 31, 41, 47, 61, 107, 181, 541, 1103, 2521\}.$$

Setting

$$N = \prod_{p \in \mathcal{P}} p = 1821895895860356790898731230,$$

the value of a and b can be replaced by any integers a and b satisfying

$$a \equiv 106276436867 \pmod{N} \quad \text{and} \quad b \equiv 35256392432 \pmod{N}.$$

As $\gcd(106276436867, N) = 31$ and $\gcd(35256392432, N) = 2$, these congruences are equivalent to taking $a = 31a'$ and $b = 2b'$ where a' and b' are integers satisfying

$$a' \equiv 3428272157 \pmod{58770835350334090028991330}$$

and

$$b' \equiv 17628196216 \pmod{910947947930178395449365615}.$$

As a direct application of D. Shiu's result, we have the following.

Theorem 3. *For every $k \in \mathbb{Z}^+$, there are k consecutive primes $p_1, p_2, \dots, p_k$ and k consecutive primes $q_1, q_2, \dots, q_k$ such that for any $i \in \{1, 2, \dots, k\}$, the numbers $a = 31p_i$ and $b = 2q_i$ satisfy $\gcd(a, b) = 1$ and have the property that the u_n defined by (1) are all composite.*

This latter result is not meant to be particularly significant but rather an indication that Shiu's work does provide information in cases where covering systems are used to form composite numbers.

Regarding open problems, given the recent excellent works surrounding the non-existence of covering systems of particular forms (cf. [1, 2, 14, 15]), the authors are not convinced that widely digitally delicate primes exist in every base. Thus, a tantalizing question is whether they exist or whether a positive proportion of the primes in every base are widely digitally delicate. In the opposite direction, as noted in [11], Carl Pomerance has asked for an unconditional proof that there exist infinitely many primes which are not digitally delicate or which are not widely digitally delicate. For other open problems in this direction, see the end of the introdutcions in [9] and [11].

2. The first steps of the argument

As noted in the introduction, to prove Theorem 1, the work of D. Shiu [26] implies that it suffices to obtain an arithmetic progression $An+B$, with A and B relatively prime positive integers, such that every prime in the arithmetic progression is widely digitally delicate. We will determine such an A and B by finding relatively prime positive integers A and B satisfying property $(*)$ given by

$(*)$ If $d \in \{-9,-8,\dots,-1\} \cup \{1,2,\dots,9\}$, then each number in the set

$$\mathcal{A}_d = \left\{An + B + d \cdot 10^k : n \in \mathbb{Z}^+, k \in \mathbb{Z}^+ \cup \{0\}\right\}$$

is composite.

As changing a digit of $An+B$, including any one of its infinitely many leading zero digits, corresponds to adding or subtracting one of the numbers $1,2,\dots,9$ from a digit of $An+B$, we see that relatively prime positive integers A and B satisfying property $(*)$ also satisfy the property we want, that every prime in $An+B$ is widely digitally delicate.

To find relatively prime positive integers A and B satisfying property $(*)$, we make use of covering systems which we define as follows.

Definition 1. *A covering system (or covering) is a finite set of congruences*

$$x \equiv a_1 \pmod{m_1}, \quad x \equiv a_2 \pmod{m_2}, \quad \dots, \quad x \equiv a_r \pmod{m_r},$$

where $r \in \mathbb{Z}^+$, each $a_j \in \mathbb{Z}$, and each $m_j \in \mathbb{Z}^+$, such that every integer satisfies at least one congruence in the set of congruences.

In other contexts in the literature, further restrictions can be made on the m_j, so we emphasize here that we want to allow for $m_j = 1$ and for repeated moduli (so that the m_j are not necessarily distinct). There will be restrictions on the m_j that will arise in the covering systems we build due to the approach we are using. We will see these as we proceed.

For each $d \in \{-9,-8,\dots,-1\} \cup \{1,2,\dots,9\}$, we will create a separate covering system to show that the elements of $\mathcal{A}_d$ in $(*)$ are composite. Table 1 indicates, for each d, the number of different congruences in the covering system corresponding to d.

Table 1: Number of congruences for each covering

d	# cong.
-9	232
-8	441
-7	1
-6	257
-5	268
-4	1

d	# cong.
-3	739
-2	289
-1	1
1	37
2	1
3	203

d	# cong.
4	26
5	1
6	19
7	137
8	1
9	4

The integers we are covering for each d are the exponents k on 10 in the definition of $\mathcal{A}_d$. In other words, we will want to view each exponent k as satisfying one of the congruences in our covering system for a given $\mathcal{A}_d$. In the end, the values of A and B will be determined by the congruences we choose for the covering systems as well as certain primes that arise in our method.

We clarify that the work on digitally delicate primes in prior work mentioned in the introduction used a partial covering of the integers k, that is a set of congruences where most but not all integers k satisfy at least one of the congruences, together with a sieve argument. The work in [11] on widely digitally delicate primes used covering systems for $d \in \{1, 2, \ldots, 9\}$ and the same approach of partial coverings and sieves for $d \in \{-9, -8, \ldots, -1\}$. The work in [9], like we will use in this paper, made use of covering systems for all $d \in \{-9, -8, \ldots, -1\} \cup \{1, 2, \ldots, 9\}$. For [9], some of the covering systems could be handled rather easily by taking advantage of the fact that we were looking for composite numbers satisfying a certain property rather than primes.

Next, we explain more precisely how we create and take advantage of a covering system for a given fixed $d \in \{-9, -8, \ldots, -1\} \cup \{1, 2, \ldots, 9\}$. We begin with a couple illustrative examples. Table 1 indicates that a number of the d are handled with just one congruence. This is accomplished by taking

$$A \equiv 0 \pmod 3 \qquad \text{and} \qquad B \equiv 1 \pmod 3.$$

Observe that each element of $\mathcal{A}_d$ in $(*)$ is divisible by 3 whenever $d \equiv 2 \pmod 3$. Thus, since A and B are positive, as long as we also have $B > 3$, the elements of $\mathcal{A}_d$ for such d are all composite, which is our goal. Note the crucial role of the order of 10 modulo the prime 3. The order is 1, and the covering system for each of these d is simply $k \equiv 0 \pmod 1$. Every integer satisfies this congruence, so it is a covering system. The modulus corresponds to the order of 10 modulo 3. Note also that we cannot use the prime 3 in an analogous way to cover another digit d because the choices for A and B, and hence the congruences on A and B above, are to be independent of d. For example, if $d = 4$, then $An + B + d \cdot 10^k \equiv 1 + 4 \equiv 2 \pmod 3$ and, hence, $An + B + d \cdot 10^k$ will not be divisible by 3.

As a second illustration, we see from Table 1 that we handle the digit $d = 9$ with 4 congruences. The congruences for $d = 9$ are

$$k \equiv 0 \pmod 2, \quad k \equiv 3 \pmod 4, \quad k \equiv 1 \pmod 8, \quad k \equiv 5 \pmod 8.$$

One easily checks that this is a covering system, that is that every integer k satisfies one of these congruences. To take advantage of this covering system, we choose a different prime p for each congruence with 10 having order modulo p equal to the modulus. We used the prime 11 with 10 of order 2, the prime 101 with 10 of order 4, the prime 73 with 10 of order 8, and the prime 137 with 10 of order 8. We take A divisible by each of these primes. For $(*)$, with $d = 9$, we want $An+B+9\cdot 10^k$ composite. For $k \equiv 0 \pmod 2$, we accomplish this by taking $B \equiv 2 \pmod{11}$ and $B > 11$ since then $An+B+9\cdot 10^k \equiv B+9 \equiv 0 \pmod{11}$. For $k \equiv 3 \pmod 4$, we accomplish this by taking $B \equiv 90 \pmod{101}$ and $B > 101$ since then $An+B+9\cdot 10^k \equiv 90+9\cdot 10^3 \equiv 9090 \equiv 0 \pmod{101}$. Similarly, for $k \equiv 1 \pmod 8$ and $B \equiv 56 \pmod{73}$, we obtain $An + B + 9 \cdot 10^k \equiv 0 \pmod{73}$; and for $k \equiv 5 \pmod 8$ and $B \equiv 90 \pmod{137}$, we obtain $An + B + 9 \cdot 10^k \equiv 0 \pmod{137}$. Thus, taking $B > 137$, we see that $(*)$ holds with $d = 9$.

Of some significance to our explanations later, we note that we could have interchanged the roles of the primes 73 and 137 since 10 has the same order for each of these primes. In other words, we could associate 137 with the congruence $k \equiv 1 \pmod 8$ above and associate 73 with the congruence $k \equiv 5 \pmod 8$. Then for $k \equiv 1 \pmod 8$ and $B \equiv 47 \pmod{137}$, we would have $An + B + 9 \cdot 10^k \equiv 0 \pmod{137}$; and for $k \equiv 5 \pmod 8$ and $B \equiv 17 \pmod{73}$, we would have $An + B + 9 \cdot 10^k \equiv 0 \pmod{73}$. In general, in our construction of widely digitally delicate primes, we want each congruence $k \equiv a \pmod m$ in a covering system associated with a prime p for which the order of 10 modulo p is m, but how we choose the ordering of those primes (which prime goes to which congruence) for a fixed modulus m is irrelevant.

For each $d \in \{-9, -8, \dots, -1\} \cup \{1, 2, \dots, 9\}$, we determine a covering system of congruences for k, where each modulus m corresponds to the order of 10 modulo some prime p. This imposes a condition on A, namely that A is divisible by each of these primes p. Fixing d, a congruence from our covering system $k \equiv a \pmod m$, and a corresponding prime p with 10 having order m modulo p, we determine B such that $An + B + d \cdot 10^k \equiv B + d \cdot 10^a \equiv 0 \pmod p$. Note that the values of d, a and p dictate the congruence condition for B modulo p. Each prime p will correspond to a unique congruence condition $B \equiv -d \cdot 10^a \pmod p$, so the Chinese Remainder Theorem implies the existence of a $B \in \mathbb{Z}^+$ simultaneously satisfying all the congruences conditions modulo primes on B. As long as B is large enough, then the condition $(*)$ will hold.

To make sure that there is a prime of the form $An+B$, we will want $\gcd(A, B) = 1$.

For $k \equiv a \pmod{m}$ and a corresponding prime p as above, we will have A divisible by p and $B \equiv -d \cdot 10^a \pmod{p}$. Since $d \in \{-9, -8, \ldots, -1\} \cup \{1, 2, \ldots, 9\}$, if $p \geq 11$, then we see that $p \nmid B$. We will not be using the primes $p \in \{2, 5\}$ as 10 does not have an order modulo these primes. We have already seen that we are using the prime $p = 3$ for $d \equiv 2 \pmod{3}$, so this ensures that $3 \nmid B$. We will use $p = 7$ for $d \in \{-9, -8, -6, -5, -3, 3, 4\}$, which then implies $7 \nmid B$. Therefore, the condition $\gcd(A, B) = 1$ will hold.

Recall that we used the same congruence and corresponding prime in our covering system for each $d \equiv 2 \pmod{3}$. There is no obstacle to repeating a congruence for different d if the corresponding prime, having 10 of order the modulus, is different. But in the case of $d \equiv 2 \pmod{3}$, the same prime 3 was used for different d. To illustrate how we can repeat the use of a prime, we return to how we used the prime $p = 11$ above for $d = 9$. We ended up with $A \equiv 0 \pmod{11}$ and $B \equiv 2 \pmod{11}$. In order for us to take advantage of the prime $p = 11$ for d, we therefore want $An + B + d \cdot 10^k \equiv 2 + d \cdot 10^k \equiv 0 \pmod{11}$. It is easy to check that this holds for $(d, k) \in \{(-9, 1), (-2, 0), (2, 1), (9, 0)\}$. The case $(d, k) = (9, 0)$ is from our example with $d = 9$ above. The case $(d, k) = (2, 1)$ does not serve a purpose for us as $d = 2$ was covered by our earlier example using the prime 3 for all $d \equiv 2 \pmod{3}$. The cases where $(d, k) \in \{(-9, 1), (-2, 0)\}$ are significant, and we make use of congruences modulo 11 in the covering systems for $d = -9$ and $d = -2$. Thus, we are able to repeat the use of some primes for different values of d. However, this is not the case for most primes we used. A complete list of the primes which we were able to use for more than one value of d is given in Table 2, together with the list of corresponding d's. The function $\rho(m, p)$ in this table will be explained in the next section.

Recalling that the modulus in a covering system is equal to the order of 10 modulo a prime p, the role of primes and the order of 10 modulo those primes is significant in coming up with covering systems to deduce $(*)$. A modulus m can be used in a given covering system as many times as there are primes with 10 of order m. Thus, for the covering system for $d = 9$, we saw the modulus 8 being used twice as there are two primes with 10 of order 8, namely the primes 73 and 137. One can look at a list of primitive prime factors of $10^k - 1$ such as in [3], but we needed much more extensive data than what is contained there. Our approach uses that the complete list of primes for which 10 has a given order m is the same as the list of primes dividing $\Phi_m(10)$ and not dividing m where $\Phi_m(x)$ is the m-th cyclotomic polynomial (cf. [3, 9, 11]). We used Magma V2.23-1 on a 2017 MacBook Pro to determine different primes dividing $\Phi_m(10)$. We did not always get a complete factorization, but used that if the remaining unfactored part of $\Phi_m(10)$ is composite, relatively prime to the factored part of $\Phi_m(10)$ and m, and not a prime power, then there must be at least two further distinct prime factors of $\Phi_m(10)$. This allowed us then to determine a lower bound on the number of

Table 2: Primes used for more than one digit d

prime	d's	$\rho(m,p)$
3	$-7,-4,-1,2,5,8$	1
7	$-9,-8,-6,-5,-3,3,4$	1
11	$-9,-2,9$	1
13	$-9,-3,3,4$	2
17	$-8,-6,-3,-2,7$	1
19	$-6,4$	1
23	$-9,-8,-6,-3,3,7$	1
29	$-9,-8,-6,1,3$	1
31	$-8,-2,6$	1
37	$3,4$	1
43	$-8,-3,1$	1
53	$-8,-5,3$	1
61	$-6,3,6$	1
67	$-9,7$	1
79	$-9,-5$	2
89	$-6,-3,7$	1
103	$-9,-8,-3$	1

prime	d's	$\rho(m,p)$
199	$-6,-3,7$	1
211	$-6,6$	1
241	$-6,6$	2
331	$-8,7$	1
353	$-6,7$	1
409	$-8,-3$	1
449	$-9,7$	2
2161	$-6,6$	3
3541	$-6,6$	1
9091	$-6,6$	1
27961	$-6,6$	2
1676321	$-6,6$	1
3762091	$-6,6$	2
4188901	$-6,6$	2
39526741	$-6,6$	3
5964848081	$-6,6$	2

distinct primes of a given order m. Though we used most of these in our coverings, sometimes we found extra primes that we did not need to use.

In total, we made use of 673 different moduli m and 2596 different primes dividing $\Phi_m(10)$ for such m. Of the 2596 different primes, there are 590 which came from 295 composite numbers arising from an unfactored part of some $\Phi_m(10)$, and there are 63 other composite numbers for which only one prime factor of each of the composite numbers was used. The largest explicit prime (not coming from the $295+63=358$ composite numbers) has 1700 digits, arising from testing what was initially a large unfactored part of $\Phi_m(10)$ for primality and determining it is a prime. The largest of the 358 composite numbers has 17234 digits. For obvious reasons, we will avoid listing these primes and composites in this paper, though to help with verification of the results, we are providing the data from our computations in [7]; more explicit tables can also be found in [18].

Table 4 in the appendix gives, for each of the 673 different moduli m, the detailed information on the number of distinct primes we used with 10 of order m, which we denote by $L(m)$. Thus, $L(m)$ is a lower bound on the total number of distinct primes with 10 of order m. Note that $L(m)$ is less than or equal to the number of distinct primes dividing $\Phi_m(10)$ but not dividing m.

For each $d \in \{-9,-8,\dots,-1\} \cup \{1,2,\dots,9\}$, the goal is to find a covering system so that $(*)$ holds. We have already given the covering systems we obtained for $d \equiv 2$

(mod 3) and for $d = 9$. In the next section and the appendix, we elaborate on the covering systems for the remaining d. We also explain how the reader can verify the data showing these covering systems satisfy the conditions needed for $(*)$.

3. Finishing the argument

To finish the argument, we need to present a covering system for each value of d in $\{-9, -8, \ldots, -1\} \cup \{1, 2, \ldots, 9\}$ as described in the previous section. For the purposes of keeping the presentation of these coverings systems manageable, for each m listed in Table 4, we take the $L(m)$ primes we found with 10 of order m and order them in some way. Corresponding to the discussion concerning $d = 9$ and the primes 73 and 137, the particular ordering is not important to us (for example, increasing order would be fine). Suppose the primes corresponding to m are ordered in some way as $p_1, p_2, \ldots, p_{L(m)}$. We define $\rho(p_j, m) = j$. Thus, if p_j is the j-th prime in our ordering of the primes with 10 of order m, we have $\rho(p_j, m) = j$. The particular values we used for $\rho(p_j, m)$ is not important to the arguments. So as to make the entries in Table 2 correct, the entries for $\rho(p, m)$ indicate the values we used for those primes. For example, Table 4 indicates there are 2 primes of order 6. One of them is 7. Table 2 indicates then that $\rho(7, 6) = 1$. Thus, we put 7 as the first of the 2 primes with 10 of order 6. The other prime with 10 of order 6 is 13, and as Table 2 indicates we set 13 as the second of the 2 primes with 10 of order 6.

Tables 5-16 give the covering systems used for each $d \in \{-9, -8, \ldots, -1\} \cup \{1, 2, \ldots, 9\}$ with $d \not\equiv 2 \pmod 3$. Rather than indicating the prime, which in some cases has thousands of digits, corresponding to each congruence $k \equiv a \pmod m$ listed, we simply wrote the value of $\rho(m, p)$. As m corresponds to the modulus used in the given congruence $k \equiv a \pmod m$ and the ordering of the primes is not significant to our arguments (any ordering will do), this is enough information to confirm the covering arguments.

That said, the time consuming task of coming up with the $L(m)$ primes to order for each m is nontrivial (at least at this point in time). So that this work does not need to be repeated, a complete list of the $L(m)$ primes for each m is given in [7]. Further, the tables in the form of lists can be found there as well, with the third column in each case replaced by the prime we used with 10 of order the modulus of the congruence in the second column. In the way of clarity, recall that the primes were not explicitly computed in the case that the unfactored part of $\Phi_m(10)$ was tested to be composite; instead the composite number is listed in place of both primes in [7].

For the remainder of this section, we clarify how to verify the information in Tables 5-16. We address both verification of the covering systems and the information on the primes as listed in [7].

3.1. Covering Verification.

The most direct way to check that a system $\mathcal{C}$ of congruences

$$x \equiv a_1 \pmod{m_1}, \quad x \equiv a_2 \pmod{m_2}, \quad \dots, \quad x \equiv a_s \pmod{m_s}$$

is a covering system is to set $\ell = \operatorname{lcm}(m_1, m_2, \dots, m_s)$ and then to check if every integer in the interval $[0, \ell-1]$ satisfies at least one congruence in $\mathcal{C}$. If not, then $\mathcal{C}$ is not a covering system. If on the other hand, every integer in $[0, \ell-1]$ satisfies a congruence in $\mathcal{C}$, then $\mathcal{C}$ is a covering system. To see the latter, let n be an arbitrary integer, and write $n = \ell q + r$ where q and r are integers with $0 \le r \le \ell - 1$. Since $r \in [0, \ell-1]$ satisfies some $x \equiv a_j \pmod{m_j}$ and since $\ell \equiv 0 \pmod{m_j}$, we deduce for this same j that $n = \ell q + r \equiv a_j \pmod{m_j}$.

The above is a satisfactory approach if ℓ is not too large. For the values of d in $\{-9, -8, \dots, -1\} \cup \{1, 2, \dots, 9\}$ with $d \not\equiv 2 \pmod 3$, the least common multiple ℓ given by the congruences in Tables 5-16 are listed in Table 3. The maximum prime divisor of ℓ is also listed in the fourth column of Table 3. The value of ℓ can exceed 10^{12}, so we found a more efficient way to test whether one of our systems $\mathcal{C}$ of congruences, where ℓ is large, is a covering system.

Table 3: Least common multiple of the moduli for the coverings in each table

d	Table	ℓ	max p
-9	5	14433138720	31
-8	6	699847948800	17
-6	7	1045044000	29
-5	8	56216160	13
-3	9	1486147703040	19
-2	10	321253732800	23

d	Table	ℓ	max p
1	11	5040	7
3	12	133333200	37
4	13	1296	3
6	14	360	5
7	15	18295200	11
9	16	8	2

Suppose $\ell > 10^6$ in Table 3, and the corresponding collection of congruences coming from the table indicated in the second column is $\mathcal{C}$. Let q be the largest prime divisor of ℓ as indicated in the fourth column. Let $w = 4 \cdot 3 \cdot 5 \cdot q$. This choice of w was selected on the basis of some trial and error; other choices are certainly reasonable. We do however want and have that w divides ℓ. Based on the comments above, we would like to know if every integer in the interval $[0, \ell-1]$ satisfies at least one congruence in $\mathcal{C}$. The basic idea is to take each $u \in [0, w-1]$ and to consider the integers that are congruent to u modulo w in $[0, \ell-1]$. One advantage of doing this is that not every congruence in $\mathcal{C}$ needs to be considered. For example, take $d = -3$. Then Table 3 indicates $\ell = 1486147703040$ and Table 1 indicates the number of congruences in $\mathcal{C}$ is 739. From Table 9, the first few of the congruences in $\mathcal{C}$ are

$$k \equiv 4 \pmod 6, \quad k \equiv 5 \pmod 6, \quad k \equiv 0 \pmod{16}, \quad k \equiv 11 \pmod{21}.$$

Here, $w = 4 \cdot 3 \cdot 5 \cdot 19 = 1140$. If we take $u = 0$, then only the third of these congruences can be satisfied by an integer k congruent to u modulo w, as each of the other ones requires $k \not\equiv 0 \pmod 3$ whereas $k \equiv u \pmod w$ requires $k \equiv 0 \pmod 3$. Let $\mathcal{C}'$ be the congruences in $\mathcal{C}$ which are consistent with $k \equiv u \pmod w$. One can determine these congruences by using that there exist integers satisfying both $k \equiv a \pmod m$ and $k \equiv u \pmod w$ if and only if $a \equiv u \pmod{\gcd(m, w)}$.

Observe that, with $u \in [0, w-1]$ fixed, we would like to know if each integer v of the form

$$v = wt + u, \qquad \text{with } 0 \le t \le (\ell/w) - 1 \tag{2}$$

satisfies at least one congruence in $\mathcal{C}'$. The main advantage of this approach is that, as we shall now see, not all ℓ/w values of t need to be considered. First, we note that if $\mathcal{C}'$ is the empty set, then the integers in (2) are not covered and therefore $\mathcal{C}$ is not a covering system. Suppose then that $|\mathcal{C}'| \ge 1$. Let ℓ' denote the least common multiple of the moduli in $\mathcal{C}'$. Let $\delta = \gcd(w, \ell')$. We claim that we need only consider $v = wt + u$ where $0 \le t \le (\ell'/\delta) - 1$. To see this, suppose we know that every $v = wt + u$ with $0 \le t \le (\ell'/\delta) - 1$ satisfies one of the congruences in $\mathcal{C}'$. There are integers q, q', r and r' satisfying $t = \ell' q' + r'$ where $0 \le r' \le \ell' - 1$ and $r' = (\ell'/\delta)q + r$, where $0 \le r \le (\ell'/\delta) - 1$. Then

$$v = wt + u = w\ell' q' + wr' + u = w\ell' q' + (w/\delta)\ell' q + wr + u.$$

The definition of δ implies that $w/\delta \in \mathbb{Z}$. As each modulus in $\mathcal{C}'$ divides ℓ', we see that v satisfies a congruence in $\mathcal{C}'$ if and only if $wr + u$ does. Here, w and u are fixed and $0 \le r \le (\ell'/\delta) - 1$. Thus, we see that for each $u \in [0, w-1]$, we can restrict to determining whether v in (2) satisfies a congruence in $\mathcal{C}'$ for $0 \le t \le (\ell'/\delta) - 1$. Returning to the example of $d = -3$, $\ell = 1486147703040$, and $|\mathcal{C}| = 739$, where $w = 1140$ and we considered $u = 0$, one can check that $|\mathcal{C}'| = 19$, $\ell' = 12640320$, $\delta = w$ and $\ell'/\delta = 11088$. Thus, what started out as ominously checking whether over 10^{12} integers each satisfy at least one of 739 different congruences is reduced in the case of $u = 0$ to looking at whether 11088 integers each satisfy at least one of 19 different congruences. As $u \in [0, w-1]$ varies, the number of computations does as well. An extreme case for $d = -3$ occurs for $u = 75$, where we get $\ell'/\delta = 14325696$ and $|\mathcal{C}'| = 47$. As d and u vary, though, this computation becomes manageable for determining that we have covering systems for each d in $\{-9, -8, \ldots, -1\} \cup \{1, 2, \ldots, 9\}$ with $d \not\equiv 2 \pmod 3$ and $\ell > 10^6$. On a 2017 MacBook Plus running Maple 2019 with a 2.3 GHz Dual-Core Intel Core i5 processor, the total cpu time for determining the systems of congruences in Tables 5-16 are all covering systems took approximately 2.9 cpu hours, with almost all of this time spent on the case $d = -3$ which took 2.7 hours. The largest value of ℓ'/δ encountered was $\ell'/\delta = 14325696$ which occurred precisely for $d = -3$ and $u \in \{75, 303, 531, 759, 987\}$.

3.2. Data check.

The most cumbersome task for us was the determination of the data in Table 4. As noted earlier, although the reader can check the data there directly, we have made the list of primes corresponding to each m available through [7]. With the list of such primes for each m, it is still worth indicating how the data can be checked. Recall, in particular, the list of primes is not explicit in the case that there was an unfactored part of $\Phi_m(10)$ that we wanted to take advantage of. In this subsection, we elaborate on what checks should be and were done. All computations below were done with the MacBook Pro mentioned at the end of the last subsection and using Magma V2.23-1.

For each modulus m used in our constructions (listed in Table 4), we made a list of primes $p_1, p_2, \ldots, p_s$, written in increasing order, together with up to two additional primes q_1 and q_2, included after p_s on the list but not written explicitly (as we will discuss). Each prime came from a factorization or partial factorization of $\Phi_m(10)$. The primes $p_1, p_2, \ldots, p_s$ are the distinct primes appearing in the factored part of $\Phi_m(10)$, and as noted earlier do not include primes dividing m. In some cases, a complete factorization was found for $\Phi_m(10)$. For such m, there are no additional primes q_1 and q_2. We note that, if $\Phi_m(10)$ was completely factored, Magma determined the primes $p_1, p_2, \ldots, p_{s-1}$ using one or more factoring routines but obtained the last prime p_s by a primality test. If $\Phi_m(10)$ had an unfactored part $Q > 1$ (already tested to be composite), then we checked that Q is relatively prime to $mp_1p_2 \cdots p_s$ and that Q is not of the form N^k with $N \in \mathbb{Z}^+$ and k an integer greater than or equal to 2. As this was always the case for the Q tested, we knew each such Q had two distinct prime factors q_1 and q_2. We deduce that there are at least two more primes q_j, $j \in \{1, 2\}$, different from $p_1, p_2, \ldots, p_s$ for which 10 has order m modulo q_j. As the data only contains the primes used in the covering systems, we only included the primes q_1 and q_2 that were used. Thus, despite Q having at least two distinct prime divisors, we may have listed anywhere from 0 to 2 of them. The question arises, however, as to how one can list primes that we do not know; there are primes q_1 and q_2 dividing Q, but we were unable to (or chose not to) factor Q to determine them explicitly. Instead of listing q_1 and q_2 then, we opted to list Q. Thus, for each m we associated a list of one of the forms

$$[p_1, p_2, \ldots, p_s], \quad [p_1, p_2, \ldots, p_s, Q], \quad [p_1, p_2, \ldots, p_s, Q, Q],$$

depending on whether Q either did not exist or we used no prime factor of Q, we used one prime factor of Q, or we used two prime factors of Q, respectively. It is possible that $s = 0$; as an unusual example, the two moduli 2888 and 2976 both have one prime associated with them coming from a composite number, so the list of primes for these take the middle form with no p_j.

For a fixed m, given such a list, say from [7], one nearly needs to check:

- Each element of the list divides $\Phi_m(10)$.
- Each element of the list is relatively prime to m.
- There is at most one composite number, say $Q > 1$, in the list, which may appear at most twice. The other numbers in the list are distinct primes.
- If the composite number Q exists, then $\gcd(Q, p_1p_2\cdots p_s) = 1$.
- If the composite number Q exists twice, then $Q^{1/k} \notin \mathbb{Z}^+$ for every integer $k \in [2, \log(Q)/\log(2)]$.

The upper bond in the last item above is simply because $k > \log(Q)/\log(2)$ implies $1 < Q^{1/k} < 2$ and, hence, $Q^{1/k}$ is not an integer. With these computations, the value of $L(m)$ in Table 4 is simply the number of elements in the list associated with m.

With the data from the tables in the Appendix, also available in [7] with the indicated primes $p_1, p_2, \ldots, p_s, q_1, q_2$ depending on m as above, some further details need to be checked to fully justify the computations. We verified that whenever m is used as a modulus in a table, it was associated with one of the primes dividing $\Phi_m(10)$. Furthermore, for any given $d \in \{-9, -8, \ldots, -1\} \cup \{1, 2, \ldots, 9\}$, the complete list of primes used as the congruences vary are distinct, noting that q_1 and q_2, for a given m, will be denoted by the same number Q but represent two distinct prime divisors of Q. As d varies, a given modulus m and a prime p dividing $\Phi_m(10)$ can be used more than once as indicated in Table 2. To elaborate, suppose such an m and p is used for each $d \in \mathcal{D} \subseteq \{-9, -8, \ldots, -1\} \cup \{1, 2, \ldots, 9\}$. For each $d \in \mathcal{D}$, then, there corresponds a congruence $k \equiv a \pmod{m}$, where $a = a(d)$ will depend on d, as well as m and p. As noted earlier, this is permissible if and only if the values of $d \cdot 10^{a(d)}$ are congruent modulo p for all $d \in \mathcal{D}$. Thus, for each p that occurs in more than one table, as in Table 2, a check is done to verify the corresponding values of $d \cdot 10^{a(d)}$ are congruent modulo p.

The verification of the covering systems needed for Theorem 1 is complete, and the work of D. Shiu [26] now implies the theorem.

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Appendix

This appendix begins with Table 4, which gives a lower bound $L(m)$ on the number of distinct prime divisors of $\Phi_m(10)$ that are relatively prime to m. The m listed correspond to moduli used in our coverings. The number $L(m)$ also provides a lower bound on the number of primes p for which 10 has order m modulo p.

After Table 4, the remaining Tables 5-16 give the congruences $k \equiv a \pmod{m}$ that form the covering systems we obtained for $d \in \{-9, -8, \ldots, -1\} \cup \{1, 2, \ldots, 9\}$ with $d \not\equiv 2 \pmod{3}$. For each congruence, there is an associated prime coming from the primes listed in Table 4 and that prime is tabulated in the second columns of Tables 5-16 (using the notation $\rho(m, p)$ discussed earlier in this paper).

Table 4: Number of primes used, $L = L(m)$, with 10 of order m (Part I)

m	L	m	L	m	L	m	L	m	L	m	L
1	1	56	2	132	3	234	3	361	7	520	2
2	1	57	3	133	3	238	3	363	3	522	4
3	1	58	2	135	5	240	3	364	3	527	4
4	1	60	3	136	2	242	5	368	2	528	3
5	2	62	1	138	3	247	5	370	2	532	5
6	2	63	3	140	5	248	5	372	3	540	7
7	2	64	4	143	4	250	2	374	5	544	3
8	2	65	2	144	2	252	2	377	4	546	2
9	1	66	2	145	4	253	5	380	8	552	4
10	1	68	3	148	6	255	4	384	5	555	6
11	2	69	3	152	4	259	5	390	3	561	4
12	1	70	2	153	6	260	5	391	4	570	4
13	3	72	3	154	6	261	8	396	3	572	5
14	1	74	3	155	3	264	7	399	5	575	6
15	2	75	3	156	3	266	3	403	5	578	5
16	2	76	2	161	5	270	4	406	5	580	4
17	2	77	4	162	4	272	2	407	6	589	2
18	2	78	4	165	3	273	5	408	5	592	2
19	1	80	2	168	3	275	5	414	5	594	5
20	2	81	5	169	3	276	5	416	5	595	4
21	3	84	2	170	3	280	2	418	4	598	3
22	3	85	3	171	3	285	3	420	4	605	2
23	1	87	3	174	3	286	5	425	3	608	3
24	1	88	2	175	3	289	3	429	3	609	3
25	3	90	2	176	2	290	6	432	2	612	6
26	2	91	7	180	3	296	3	434	5	620	2
27	2	92	3	182	2	297	6	435	4	621	4
28	3	93	1	184	2	299	3	437	5	624	2
29	5	95	5	185	4	304	3	440	2	627	4
30	3	96	4	186	4	306	6	442	5	630	5
31	3	99	4	187	3	310	4	444	4	638	5
32	2	100	4	190	3	312	5	455	5	644	3
33	2	102	2	192	4	315	3	456	5	646	7
34	3	104	2	195	3	319	4	459	5	651	3
35	3	105	3	198	2	322	6	460	8	660	8
36	1	108	3	203	4	323	3	462	3	663	3
37	3	110	4	204	6	330	3	464	7	665	3
38	1	111	3	207	3	333	4	465	5	666	4
39	1	112	2	208	6	336	3	476	5	672	6
40	2	114	2	209	5	338	4	480	3	676	3
42	3	115	5	210	3	340	5	483	7	680	4
44	2	116	4	216	2	341	5	484	6	682	6
45	2	117	4	217	3	342	5	494	4	684	4
46	4	119	4	220	6	345	6	495	6	690	5
48	1	120	1	221	3	348	5	496	4	693	6
50	3	121	4	222	4	350	2	506	3	696	3
51	4	124	2	228	7	351	4	507	7	702	4
52	2	125	4	230	2	352	4	510	5	704	4
54	2	126	2	231	6	357	6	513	2	714	2
55	4	130	2	232	5	360	3	518	2	715	3

Table 4: Number of primes used, $L = L(m)$, with 10 of order m (Part II)

m	L	m	L	m	L	m	L	m	L	m	L
720	2	969	4	1288	7	1767	4	2420	6	3468	4
722	3	988	3	1292	3	1768	4	2432	3	3480	3
726	4	990	4	1302	4	1776	4	2442	3	3496	6
728	2	992	4	1311	3	1785	2	2448	2	3534	4
740	10	1001	4	1320	8	1794	6	2484	2	3549	3
741	5	1012	3	1326	2	1805	5	2508	2	3570	6
744	7	1014	4	1330	3	1824	5	2535	4	3627	5
748	4	1015	7	1332	6	1848	4	2550	2	3648	2
754	2	1020	7	1352	8	1850	5	2565	4	3696	4
759	3	1023	4	1368	4	1860	8	2576	3	3700	2
760	2	1026	4	1380	9	1862	1	2584	8	3720	2
765	2	1035	3	1386	3	1870	2	2601	4	3724	6
768	6	1036	2	1392	3	1885	3	2604	5	3740	4
777	5	1040	4	1395	5	1904	5	2622	6	3770	6
782	6	1044	6	1428	2	1932	4	2652	4	3808	2
792	2	1045	3	1430	4	1938	5	2660	2	3876	4
798	3	1054	2	1440	4	1953	6	2664	4	3960	6
805	4	1056	3	1444	4	1976	4	2704	4	3990	2
806	3	1064	3	1445	6	1980	8	2736	3	4004	2
812	6	1083	7	1452	1	1995	4	2775	7	4046	3
814	5	1085	3	1456	6	2002	5	2790	1	4060	6
816	4	1088	6	1480	5	2023	3	2793	4	4080	4
828	2	1104	7	1482	2	2024	2	2888	1	4104	2
833	5	1105	4	1488	3	2030	4	2890	3	4180	6
836	3	1110	2	1496	4	2040	5	2904	2	4224	2
840	5	1122	2	1508	4	2046	3	2907	3	4256	4
845	2	1131	4	1521	1	2052	1	2912	3	4332	4
850	1	1140	11	1530	6	2070	6	2960	5	4352	3
858	5	1150	2	1547	3	2090	3	2964	3	4356	3
867	8	1156	5	1554	6	2108	1	2976	1	4370	5
868	6	1160	2	1564	5	2112	5	3003	3	4416	6
870	7	1173	4	1581	6	2128	4	3042	4	4420	6
874	5	1178	5	1584	2	2166	5	3060	7	4440	5
880	5	1183	4	1596	4	2170	5	3094	4	4560	4
884	4	1188	2	1610	6	2176	2	3108	1	4641	2
888	4	1190	6	1612	5	2185	5	3128	3	4692	2
897	3	1196	7	1615	2	2208	7	3135	9	4752	3
910	5	1197	3	1632	4	2210	3	3162	2	4788	4
912	2	1209	3	1665	3	2220	5	3179	5	4836	5
918	4	1210	3	1666	4	2262	2	3192	3	4864	4
920	2	1216	9	1672	4	2280	4	3230	3	4896	4
924	5	1221	6	1680	4	2300	4	3249	3	5005	3
925	9	1224	5	1690	4	2312	5	3255	4	5016	6
928	4	1235	5	1700	6	2346	4	3264	4	5070	2
930	4	1240	6	1716	4	2356	3	3315	5	5130	2
931	7	1242	4	1734	6	2366	4	3330	6	5168	2
935	5	1254	5	1736	2	2380	4	3344	4	5202	4
952	6	1260	7	1740	9	2392	2	3380	4	5320	2
960	2	1275	4	1748	5	2394	5	3420	5	5328	4
966	3	1276	3	1760	2	2418	5	3432	2	5472	7

Table 4: Number of primes used, $L = L(m)$, with 10 of order m (Part III)

m	L	m	L	m	L	m	L	m	L	m	L
5544	3	6270	3	7068	5	9405	6	12540	4	25080	1
5550	6	6324	3	7254	2	9537	4	12716	5	25432	6
5586	4	6358	3	7392	4	9576	5	12996	8	25992	6
5776	2	6384	2	7448	7	9792	4	13056	2	30030	6
5780	10	6460	6	7752	5	10010	4	14508	4	37620	2
5808	4	6498	3	7980	10	10032	4	15015	3	51984	2
5814	3	6510	3	8008	4	10140	10	15960	4	60060	6
5928	2	6528	3	8092	3	10260	5	16184	4	75240	6
5985	3	6630	3	8208	4	10336	4	16720	2		
6006	6	6660	3	8360	3	10944	4	18810	3		
6069	2	6840	5	8664	3	11100	4	19074	2		
6188	3	6936	3	8704	2	11172	4	20064	4		
6256	3	6960	2	8880	3	12512	2	22344	4		

Table 5: Covering information for $d = -9$ (Part I)

congruence	p	congruence	p	congruence	p
$k \equiv 1 \pmod{2}$	1	$k \equiv 44 \pmod{744}$	2	$k \equiv 80 \pmod{465}$	4
$k \equiv 0 \pmod{6}$	2	$k \equiv 200 \pmod{744}$	4	$k \equiv 266 \pmod{465}$	5
$k \equiv 4 \pmod{6}$	1	$k \equiv 386 \pmod{744}$	3	$k \equiv 452 \pmod{930}$	1
$k \equiv 6 \pmod{32}$	2	$k \equiv 572 \pmod{744}$	5	$k \equiv 638 \pmod{930}$	2
$k \equiv 26 \pmod{28}$	1	$k \equiv 728 \pmod{744}$	6	$k \equiv 824 \pmod{930}$	3
$k \equiv 16 \pmod{22}$	1	$k \equiv 170 \pmod{744}$	7	$k \equiv 50 \pmod{930}$	4
$k \equiv 26 \pmod{33}$	1	$k \equiv 414 \pmod{496}$	1	$k \equiv 236 \pmod{620}$	1
$k \equiv 9 \pmod{13}$	2	$k \equiv 446 \pmod{496}$	2	$k \equiv 546 \pmod{620}$	2
$k \equiv 0 \pmod{31}$	1	$k \equiv 478 \pmod{496}$	3	$k \equiv 112 \pmod{1240}$	1
$k \equiv 1 \pmod{31}$	2	$k \equiv 14 \pmod{496}$	4	$k \equiv 1042 \pmod{1240}$	2
$k \equiv 2 \pmod{31}$	3	$k \equiv 662 \pmod{992}$	1	$k \equiv 732 \pmod{1240}$	4
$k \equiv 34 \pmod{62}$	1	$k \equiv 694 \pmod{992}$	2	$k \equiv 422 \pmod{1240}$	3
$k \equiv 35 \pmod{93}$	1	$k \equiv 726 \pmod{992}$	3	$k \equiv 608 \pmod{1240}$	5
$k \equiv 98 \pmod{186}$	1	$k \equiv 758 \pmod{992}$	4	$k \equiv 1228 \pmod{1240}$	6
$k \equiv 68 \pmod{186}$	2	$k \equiv 356 \pmod{1488}$	1	$k \equiv 1538 \pmod{1860}$	1
$k \equiv 38 \pmod{186}$	3	$k \equiv 1100 \pmod{1488}$	2	$k \equiv 1724 \pmod{1860}$	2
$k \equiv 8 \pmod{186}$	4	$k \equiv 542 \pmod{1488}$	3	$k \equiv 794 \pmod{1860}$	4
$k \equiv 40 \pmod{124}$	1	$k \equiv 2774 \pmod{2976}$	1	$k \equiv 20 \pmod{1860}$	3
$k \equiv 102 \pmod{124}$	2	$k \equiv 140 \pmod{155}$	1	$k \equiv 950 \pmod{1860}$	5
$k \equiv 320 \pmod{372}$	1	$k \equiv 16 \pmod{155}$	2	$k \equiv 1136 \pmod{1860}$	6
$k \equiv 134 \pmod{372}$	2	$k \equiv 47 \pmod{155}$	3	$k \equiv 206 \pmod{1860}$	7
$k \equiv 104 \pmod{372}$	3	$k \equiv 78 \pmod{310}$	1	$k \equiv 392 \pmod{1860}$	8
$k \equiv 42 \pmod{248}$	1	$k \equiv 264 \pmod{310}$	2	$k \equiv 1322 \pmod{3720}$	1
$k \equiv 136 \pmod{248}$	2	$k \equiv 110 \pmod{310}$	3	$k \equiv 3182 \pmod{3720}$	2
$k \equiv 74 \pmod{248}$	3	$k \equiv 296 \pmod{310}$	4	$k \equiv 578 \pmod{1395}$	1
$k \equiv 12 \pmod{248}$	4	$k \equiv 17 \pmod{465}$	1	$k \equiv 113 \pmod{1395}$	2
$k \equiv 168 \pmod{248}$	5	$k \equiv 203 \pmod{465}$	2	$k \equiv 1043 \pmod{1395}$	3
$k \equiv 602 \pmod{744}$	1	$k \equiv 389 \pmod{465}$	3	$k \equiv 299 \pmod{1395}$	4

Table 5: Covering information for $d = -9$ (Part II)

congruence	p
$k \equiv 1229 \pmod{1395}$	5
$k \equiv 764 \pmod{2790}$	1
$k \equiv 21 \pmod{217}$	1
$k \equiv 176 \pmod{217}$	2
$k \equiv 114 \pmod{217}$	3
$k \equiv 52 \pmod{434}$	1
$k \equiv 424 \pmod{434}$	2
$k \equiv 300 \pmod{434}$	3
$k \equiv 84 \pmod{434}$	4
$k \equiv 22 \pmod{434}$	5
$k \equiv 611 \pmod{651}$	1
$k \equiv 332 \pmod{651}$	2
$k \equiv 53 \pmod{651}$	3
$k \equiv 146 \pmod{1302}$	1
$k \equiv 1232 \pmod{1302}$	2
$k \equiv 302 \pmod{1302}$	3
$k \equiv 674 \pmod{1302}$	4
$k \equiv 796 \pmod{868}$	1
$k \equiv 208 \pmod{868}$	2
$k \equiv 488 \pmod{868}$	3
$k \equiv 768 \pmod{868}$	4
$k \equiv 612 \pmod{868}$	5
$k \equiv 178 \pmod{868}$	6
$k \equiv 116 \pmod{2604}$	1
$k \equiv 1418 \pmod{2604}$	2
$k \equiv 860 \pmod{2604}$	3
$k \equiv 2162 \pmod{2604}$	4
$k \equiv 644 \pmod{2604}$	5
$k \equiv 210 \pmod{1736}$	1
$k \equiv 1078 \pmod{1736}$	2
$k \equiv 365 \pmod{1085}$	1
$k \equiv 1016 \pmod{1085}$	2
$k \equiv 582 \pmod{1085}$	3
$k \equiv 148 \pmod{2170}$	1
$k \equiv 1884 \pmod{2170}$	2
$k \equiv 520 \pmod{2170}$	3
$k \equiv 86 \pmod{2170}$	4
$k \equiv 1822 \pmod{2170}$	5
$k \equiv 1388 \pmod{3255}$	1
$k \equiv 2039 \pmod{3255}$	2
$k \equiv 1760 \pmod{3255}$	3
$k \equiv 2411 \pmod{3255}$	4
$k \equiv 3062 \pmod{6510}$	1
$k \equiv 458 \pmod{6510}$	2
$k \equiv 4364 \pmod{6510}$	3
$k \equiv 830 \pmod{1953}$	1
$k \equiv 1481 \pmod{1953}$	2
$k \equiv 179 \pmod{1953}$	3
$k \equiv 272 \pmod{1953}$	4
$k \equiv 923 \pmod{1953}$	5

congruence	p
$k \equiv 1574 \pmod{1953}$	6
$k \equiv 242 \pmod{341}$	1
$k \equiv 56 \pmod{341}$	2
$k \equiv 211 \pmod{341}$	3
$k \equiv 25 \pmod{341}$	4
$k \equiv 149 \pmod{341}$	5
$k \equiv 304 \pmod{682}$	1
$k \equiv 118 \pmod{682}$	2
$k \equiv 614 \pmod{682}$	3
$k \equiv 428 \pmod{682}$	4
$k \equiv 88 \pmod{682}$	5
$k \equiv 584 \pmod{682}$	6
$k \equiv 398 \pmod{1023}$	1
$k \equiv 212 \pmod{1023}$	2
$k \equiv 677 \pmod{1023}$	3
$k \equiv 491 \pmod{1023}$	4
$k \equiv 1328 \pmod{2046}$	1
$k \equiv 1142 \pmod{2046}$	2
$k \equiv 956 \pmod{2046}$	3
$k \equiv 182 \pmod{403}$	1
$k \equiv 27 \pmod{403}$	2
$k \equiv 275 \pmod{403}$	3
$k \equiv 120 \pmod{403}$	4
$k \equiv 368 \pmod{403}$	5
$k \equiv 616 \pmod{806}$	1
$k \equiv 58 \pmod{806}$	2
$k \equiv 306 \pmod{806}$	3
$k \equiv 554 \pmod{1209}$	1
$k \equiv 647 \pmod{1209}$	2
$k \equiv 89 \pmod{1209}$	3
$k \equiv 740 \pmod{2418}$	1
$k \equiv 338 \pmod{2418}$	2
$k \equiv 2198 \pmod{2418}$	3
$k \equiv 1640 \pmod{2418}$	4
$k \equiv 1082 \pmod{2418}$	5
$k \equiv 524 \pmod{1612}$	1
$k \equiv 772 \pmod{1612}$	2
$k \equiv 1020 \pmod{1612}$	3
$k \equiv 1268 \pmod{1612}$	4
$k \equiv 1516 \pmod{1612}$	5
$k \equiv 2942 \pmod{4836}$	1
$k \equiv 4802 \pmod{4836}$	2
$k \equiv 1826 \pmod{4836}$	3
$k \equiv 3686 \pmod{4836}$	4
$k \equiv 710 \pmod{4836}$	5
$k \equiv 803 \pmod{3627}$	1
$k \equiv 2012 \pmod{3627}$	2
$k \equiv 3221 \pmod{3627}$	3
$k \equiv 245 \pmod{3627}$	4
$k \equiv 1454 \pmod{3627}$	5

congruence	p
$k \equiv 6290 \pmod{7254}$	1
$k \equiv 3314 \pmod{7254}$	2
$k \equiv 896 \pmod{14508}$	1
$k \equiv 8150 \pmod{14508}$	2
$k \equiv 5732 \pmod{14508}$	3
$k \equiv 12986 \pmod{14508}$	4
$k \equiv 10 \pmod{34}$	1
$k \equiv 153 \pmod{527}$	1
$k \equiv 494 \pmod{527}$	2
$k \equiv 308 \pmod{527}$	3
$k \equiv 122 \pmod{527}$	4
$k \equiv 990 \pmod{1054}$	1
$k \equiv 804 \pmod{1054}$	2
$k \equiv 1145 \pmod{1581}$	1
$k \equiv 959 \pmod{1581}$	2
$k \equiv 773 \pmod{1581}$	3
$k \equiv 587 \pmod{1581}$	4
$k \equiv 215 \pmod{1581}$	5
$k \equiv 29 \pmod{1581}$	6
$k \equiv 1424 \pmod{3162}$	1
$k \equiv 1238 \pmod{3162}$	2
$k \equiv 1052 \pmod{2108}$	1
$k \equiv 4214 \pmod{6324}$	1
$k \equiv 4028 \pmod{6324}$	2
$k \equiv 866 \pmod{6324}$	3
$k \equiv 247 \pmod{589}$	1
$k \equiv 495 \pmod{589}$	2
$k \equiv 154 \pmod{1178}$	1
$k \equiv 402 \pmod{1178}$	2
$k \equiv 650 \pmod{1178}$	3
$k \equiv 898 \pmod{1178}$	4
$k \equiv 1146 \pmod{1178}$	5
$k \equiv 1394 \pmod{1767}$	1
$k \equiv 464 \pmod{1767}$	2
$k \equiv 1301 \pmod{1767}$	3
$k \equiv 371 \pmod{1767}$	4
$k \equiv 1208 \pmod{3534}$	1
$k \equiv 278 \pmod{3534}$	2
$k \equiv 2882 \pmod{3534}$	3
$k \equiv 1952 \pmod{3534}$	4
$k \equiv 2200 \pmod{2356}$	1
$k \equiv 1022 \pmod{2356}$	2
$k \equiv 92 \pmod{2356}$	3
$k \equiv 3626 \pmod{7068}$	1
$k \equiv 2696 \pmod{7068}$	2
$k \equiv 6230 \pmod{7068}$	3
$k \equiv 5300 \pmod{7068}$	4
$k \equiv 1766 \pmod{7068}$	5

Table 6: Covering information for $d = -8$ (Part I)

congruence	p
$k \equiv 2 \pmod{13}$	1
$k \equiv 3 \pmod{21}$	1
$k \equiv 19 \pmod{28}$	1
$k \equiv 10 \pmod{22}$	1
$k \equiv 9 \pmod{110}$	1
$k \equiv 0 \pmod{6}$	1
$k \equiv 11 \pmod{15}$	1
$k \equiv 13 \pmod{16}$	1
$k \equiv 0 \pmod{17}$	1
$k \equiv 1 \pmod{17}$	2
$k \equiv 2 \pmod{34}$	1
$k \equiv 19 \pmod{34}$	2
$k \equiv 3 \pmod{34}$	3
$k \equiv 88 \pmod{204}$	1
$k \equiv 20 \pmod{204}$	2
$k \equiv 190 \pmod{204}$	3
$k \equiv 122 \pmod{204}$	4
$k \equiv 4 \pmod{68}$	1
$k \equiv 21 \pmod{68}$	2
$k \equiv 38 \pmod{68}$	3
$k \equiv 123 \pmod{136}$	1
$k \equiv 55 \pmod{136}$	2
$k \equiv 5 \pmod{272}$	1
$k \equiv 243 \pmod{272}$	2
$k \equiv 651 \pmod{1632}$	4
$k \equiv 2827 \pmod{3264}$	1
$k \equiv 1195 \pmod{3264}$	2
$k \equiv 1739 \pmod{3264}$	3
$k \equiv 107 \pmod{3264}$	4
$k \equiv 3099 \pmod{6528}$	1
$k \equiv 1467 \pmod{6528}$	2
$k \equiv 6363 \pmod{6528}$	3
$k \equiv 4731 \pmod{13056}$	1
$k \equiv 11259 \pmod{13056}$	2
$k \equiv 379 \pmod{4896}$	1
$k \equiv 2011 \pmod{4896}$	2
$k \equiv 3643 \pmod{4896}$	3
$k \equiv 4187 \pmod{4896}$	4
$k \equiv 923 \pmod{9792}$	1
$k \equiv 7451 \pmod{9792}$	2
$k \equiv 5819 \pmod{9792}$	3
$k \equiv 2555 \pmod{9792}$	4
$k \equiv 39 \pmod{544}$	1
$k \equiv 311 \pmod{544}$	2
$k \equiv 175 \pmod{544}$	3
$k \equiv 1025 \pmod{8704}$	1
$k \equiv 5377 \pmod{8704}$	2
$k \equiv 2113 \pmod{4352}$	1
$k \equiv 3201 \pmod{4352}$	2
$k \equiv 4289 \pmod{4352}$	3

congruence	p
$k \equiv 1569 \pmod{2176}$	1
$k \equiv 481 \pmod{2176}$	2
$k \equiv 73 \pmod{1088}$	1
$k \equiv 209 \pmod{1088}$	2
$k \equiv 345 \pmod{1088}$	3
$k \equiv 617 \pmod{1088}$	4
$k \equiv 753 \pmod{1088}$	5
$k \equiv 889 \pmod{1088}$	6
$k \equiv 56 \pmod{459}$	1
$k \equiv 413 \pmod{459}$	2
$k \equiv 311 \pmod{459}$	3
$k \equiv 209 \pmod{459}$	4
$k \equiv 107 \pmod{459}$	5
$k \equiv 464 \pmod{918}$	1
$k \equiv 362 \pmod{918}$	2
$k \equiv 260 \pmod{918}$	3
$k \equiv 158 \pmod{918}$	4
$k \equiv 124 \pmod{204}$	5
$k \equiv 22 \pmod{204}$	6
$k \equiv 447 \pmod{1632}$	1
$k \equiv 991 \pmod{1632}$	2
$k \equiv 1535 \pmod{1632}$	3
$k \equiv 40 \pmod{51}$	1
$k \equiv 23 \pmod{51}$	2
$k \equiv 7 \pmod{51}$	3
$k \equiv 41 \pmod{51}$	4
$k \equiv 57 \pmod{102}$	1
$k \equiv 75 \pmod{102}$	2
$k \equiv 127 \pmod{153}$	1
$k \equiv 110 \pmod{153}$	2
$k \equiv 76 \pmod{153}$	3
$k \equiv 59 \pmod{153}$	4
$k \equiv 25 \pmod{153}$	5
$k \equiv 8 \pmod{153}$	6
$k \equiv 297 \pmod{306}$	1
$k \equiv 93 \pmod{306}$	2
$k \equiv 195 \pmod{306}$	3
$k \equiv 145 \pmod{408}$	1
$k \equiv 43 \pmod{408}$	2
$k \equiv 349 \pmod{408}$	3
$k \equiv 247 \pmod{408}$	4
$k \equiv 128 \pmod{408}$	5
$k \equiv 434 \pmod{816}$	1
$k \equiv 26 \pmod{816}$	2
$k \equiv 740 \pmod{816}$	3
$k \equiv 332 \pmod{816}$	4
$k \equiv 230 \pmod{2040}$	1
$k \equiv 1046 \pmod{2040}$	2
$k \equiv 1862 \pmod{2040}$	3
$k \equiv 638 \pmod{2040}$	4

congruence	p
$k \equiv 1454 \pmod{2040}$	5
$k \equiv 111 \pmod{612}$	1
$k \equiv 9 \pmod{612}$	2
$k \equiv 519 \pmod{612}$	3
$k \equiv 417 \pmod{612}$	4
$k \equiv 315 \pmod{612}$	5
$k \equiv 213 \pmod{612}$	6
$k \equiv 94 \pmod{306}$	4
$k \equiv 298 \pmod{306}$	5
$k \equiv 196 \pmod{306}$	6
$k \equiv 77 \pmod{1224}$	1
$k \equiv 1097 \pmod{1224}$	2
$k \equiv 893 \pmod{1224}$	3
$k \equiv 689 \pmod{1224}$	4
$k \equiv 485 \pmod{1224}$	5
$k \equiv 1505 \pmod{2448}$	1
$k \equiv 281 \pmod{2448}$	2
$k \equiv 995 \pmod{1020}$	1
$k \equiv 791 \pmod{1020}$	2
$k \equiv 587 \pmod{1020}$	3
$k \equiv 383 \pmod{1020}$	4
$k \equiv 179 \pmod{1020}$	5
$k \equiv 10 \pmod{85}$	1
$k \equiv 61 \pmod{85}$	2
$k \equiv 27 \pmod{85}$	3
$k \equiv 78 \pmod{170}$	1
$k \equiv 163 \pmod{170}$	2
$k \equiv 44 \pmod{170}$	3
$k \equiv 129 \pmod{680}$	1
$k \equiv 299 \pmod{680}$	2
$k \equiv 469 \pmod{680}$	3
$k \equiv 639 \pmod{680}$	4
$k \equiv 215 \pmod{255}$	1
$k \equiv 62 \pmod{255}$	2
$k \equiv 113 \pmod{255}$	3
$k \equiv 164 \pmod{255}$	4
$k \equiv 45 \pmod{510}$	1
$k \equiv 351 \pmod{510}$	2
$k \equiv 147 \pmod{510}$	3
$k \equiv 453 \pmod{510}$	4
$k \equiv 249 \pmod{510}$	5
$k \equiv 640 \pmod{1020}$	6
$k \equiv 385 \pmod{1020}$	7
$k \equiv 130 \pmod{4080}$	2
$k \equiv 3190 \pmod{4080}$	1
$k \equiv 2170 \pmod{4080}$	3
$k \equiv 1150 \pmod{4080}$	4
$k \equiv 2935 \pmod{3060}$	2
$k \equiv 895 \pmod{3060}$	1
$k \equiv 1915 \pmod{3060}$	3

Table 6: Covering information for $d = -8$ (Part II)

congruence	p
$k \equiv 181 \pmod{765}$	1
$k \equiv 436 \pmod{765}$	2
$k \equiv 1456 \pmod{1530}$	1
$k \equiv 691 \pmod{1530}$	2
$k \equiv 1252 \pmod{1530}$	4
$k \equiv 487 \pmod{1530}$	3
$k \equiv 742 \pmod{1530}$	5
$k \equiv 1507 \pmod{1530}$	6
$k \equiv 232 \pmod{3060}$	4
$k \equiv 997 \pmod{3060}$	6
$k \equiv 1762 \pmod{3060}$	5
$k \equiv 2527 \pmod{3060}$	7
$k \equiv 28 \pmod{425}$	1
$k \equiv 283 \pmod{425}$	2
$k \equiv 113 \pmod{425}$	3
$k \equiv 368 \pmod{1700}$	1
$k \equiv 793 \pmod{1700}$	2
$k \equiv 1218 \pmod{1700}$	3
$k \equiv 1643 \pmod{1700}$	4
$k \equiv 1048 \pmod{1700}$	5
$k \equiv 198 \pmod{1700}$	6
$k \equiv 623 \pmod{850}$	1
$k \equiv 79 \pmod{1275}$	1
$k \equiv 334 \pmod{1275}$	2
$k \equiv 589 \pmod{1275}$	3
$k \equiv 844 \pmod{1275}$	4
$k \equiv 2374 \pmod{2550}$	1
$k \equiv 1099 \pmod{2550}$	2
$k \equiv 63 \pmod{119}$	1
$k \equiv 29 \pmod{119}$	2
$k \equiv 114 \pmod{119}$	3
$k \equiv 80 \pmod{119}$	4
$k \equiv 46 \pmod{238}$	1
$k \equiv 165 \pmod{238}$	2
$k \equiv 12 \pmod{238}$	3
$k \equiv 131 \pmod{476}$	1
$k \equiv 216 \pmod{476}$	2
$k \equiv 97 \pmod{476}$	3
$k \equiv 454 \pmod{476}$	4
$k \equiv 335 \pmod{476}$	5
$k \equiv 1797 \pmod{1904}$	1
$k \equiv 1321 \pmod{1904}$	2
$k \equiv 2273 \pmod{3808}$	1
$k \equiv 369 \pmod{3808}$	2
$k \equiv 336 \pmod{952}$	1
$k \equiv 217 \pmod{952}$	2
$k \equiv 98 \pmod{952}$	3
$k \equiv 931 \pmod{952}$	4
$k \equiv 812 \pmod{952}$	5
$k \equiv 574 \pmod{952}$	6

congruence	p
$k \equiv 693 \pmod{1904}$	3
$k \equiv 455 \pmod{1904}$	4
$k \equiv 1407 \pmod{1904}$	5
$k \equiv 64 \pmod{357}$	1
$k \equiv 302 \pmod{357}$	2
$k \equiv 268 \pmod{357}$	3
$k \equiv 149 \pmod{357}$	4
$k \equiv 115 \pmod{357}$	5
$k \equiv 353 \pmod{357}$	6
$k \equiv 183 \pmod{714}$	1
$k \equiv 387 \pmod{714}$	2
$k \equiv 1305 \pmod{1428}$	1
$k \equiv 591 \pmod{1428}$	2
$k \equiv 200 \pmod{595}$	1
$k \equiv 557 \pmod{595}$	2
$k \equiv 438 \pmod{595}$	3
$k \equiv 319 \pmod{595}$	4
$k \equiv 676 \pmod{1785}$	1
$k \equiv 81 \pmod{1785}$	2
$k \equiv 880 \pmod{1190}$	2
$k \equiv 761 \pmod{1190}$	1
$k \equiv 642 \pmod{1190}$	3
$k \equiv 523 \pmod{1190}$	4
$k \equiv 404 \pmod{1190}$	5
$k \equiv 285 \pmod{1190}$	6
$k \equiv 166 \pmod{2380}$	1
$k \equiv 1356 \pmod{2380}$	2
$k \equiv 47 \pmod{2380}$	3
$k \equiv 1237 \pmod{2380}$	4
$k \equiv 1118 \pmod{3570}$	1
$k \equiv 3498 \pmod{3570}$	2
$k \equiv 2308 \pmod{3570}$	3
$k \equiv 999 \pmod{3570}$	4
$k \equiv 3379 \pmod{3570}$	5
$k \equiv 2189 \pmod{3570}$	6
$k \equiv 251 \pmod{833}$	1
$k \equiv 13 \pmod{833}$	2
$k \equiv 608 \pmod{833}$	3
$k \equiv 370 \pmod{833}$	4
$k \equiv 132 \pmod{833}$	5
$k \equiv 1560 \pmod{1666}$	1
$k \equiv 727 \pmod{1666}$	2
$k \equiv 1322 \pmod{1666}$	3
$k \equiv 489 \pmod{1666}$	4
$k \equiv 99 \pmod{187}$	1
$k \equiv 133 \pmod{187}$	2
$k \equiv 167 \pmod{187}$	3
$k \equiv 14 \pmod{374}$	1
$k \equiv 201 \pmod{374}$	2
$k \equiv 48 \pmod{374}$	3

congruence	p
$k \equiv 235 \pmod{374}$	4
$k \equiv 65 \pmod{374}$	5
$k \equiv 456 \pmod{748}$	1
$k \equiv 269 \pmod{748}$	2
$k \equiv 82 \pmod{748}$	3
$k \equiv 643 \pmod{748}$	4
$k \equiv 592 \pmod{1496}$	1
$k \equiv 218 \pmod{1496}$	2
$k \equiv 1340 \pmod{1496}$	3
$k \equiv 966 \pmod{1496}$	4
$k \equiv 405 \pmod{1870}$	1
$k \equiv 31 \pmod{1870}$	2
$k \equiv 3397 \pmod{3740}$	1
$k \equiv 1527 \pmod{3740}$	2
$k \equiv 1153 \pmod{3740}$	3
$k \equiv 3023 \pmod{3740}$	4
$k \equiv 490 \pmod{935}$	1
$k \equiv 116 \pmod{935}$	2
$k \equiv 677 \pmod{935}$	3
$k \equiv 303 \pmod{935}$	4
$k \equiv 864 \pmod{935}$	5
$k \equiv 337 \pmod{561}$	1
$k \equiv 524 \pmod{561}$	2
$k \equiv 184 \pmod{561}$	3
$k \equiv 371 \pmod{561}$	4
$k \equiv 711 \pmod{1122}$	1
$k \equiv 1119 \pmod{1122}$	2
$k \equiv 117 \pmod{221}$	1
$k \equiv 66 \pmod{221}$	2
$k \equiv 185 \pmod{221}$	3
$k \equiv 134 \pmod{442}$	1
$k \equiv 355 \pmod{442}$	2
$k \equiv 304 \pmod{442}$	3
$k \equiv 83 \pmod{442}$	4
$k \equiv 32 \pmod{442}$	5
$k \equiv 1137 \pmod{1768}$	2
$k \equiv 1579 \pmod{1768}$	1
$k \equiv 253 \pmod{1768}$	3
$k \equiv 695 \pmod{1768}$	4
$k \equiv 644 \pmod{884}$	1
$k \equiv 865 \pmod{884}$	2
$k \equiv 202 \pmod{884}$	3
$k \equiv 423 \pmod{884}$	4
$k \equiv 151 \pmod{663}$	1
$k \equiv 593 \pmod{663}$	2
$k \equiv 100 \pmod{663}$	3
$k \equiv 1035 \pmod{1326}$	1
$k \equiv 321 \pmod{1326}$	2
$k \equiv 1868 \pmod{2652}$	1
$k \equiv 1205 \pmod{2652}$	2

Table 6: Covering information for $d = -8$ (Part III)

congruence	p
$k \equiv 542 \pmod{2652}$	3
$k \equiv 2531 \pmod{2652}$	4
$k \equiv 270 \pmod{1105}$	1
$k \equiv 712 \pmod{1105}$	2
$k \equiv 933 \pmod{1105}$	3
$k \equiv 49 \pmod{1105}$	4
$k \equiv 2701 \pmod{3315}$	1
$k \equiv 1596 \pmod{3315}$	2
$k \equiv 2871 \pmod{3315}$	3
$k \equiv 661 \pmod{3315}$	4
$k \equiv 1324 \pmod{3315}$	5
$k \equiv 440 \pmod{2210}$	1
$k \equiv 1545 \pmod{2210}$	2
$k \equiv 882 \pmod{2210}$	3
$k \equiv 4197 \pmod{4420}$	1
$k \equiv 1987 \pmod{4420}$	2
$k \equiv 2208 \pmod{4420}$	5
$k \equiv 3313 \pmod{4420}$	3
$k \equiv 4418 \pmod{4420}$	4
$k \equiv 1103 \pmod{4420}$	6
$k \equiv 219 \pmod{6630}$	1
$k \equiv 5744 \pmod{6630}$	2
$k \equiv 2429 \pmod{6630}$	3
$k \equiv 168 \pmod{1547}$	1
$k \equiv 610 \pmod{1547}$	2
$k \equiv 1052 \pmod{1547}$	3
$k \equiv 1936 \pmod{3094}$	1
$k \equiv 389 \pmod{3094}$	2
$k \equiv 2820 \pmod{3094}$	3
$k \equiv 1273 \pmod{3094}$	4
$k \equiv 5472 \pmod{6188}$	1
$k \equiv 3925 \pmod{6188}$	2
$k \equiv 2378 \pmod{6188}$	3
$k \equiv 4588 \pmod{4641}$	1
$k \equiv 3041 \pmod{4641}$	2
$k \equiv 16 \pmod{289}$	1
$k \equiv 33 \pmod{289}$	2
$k \equiv 50 \pmod{289}$	3
$k \equiv 356 \pmod{578}$	1
$k \equiv 67 \pmod{578}$	2
$k \equiv 84 \pmod{578}$	3
$k \equiv 373 \pmod{578}$	4
$k \equiv 390 \pmod{578}$	5
$k \equiv 1257 \pmod{2312}$	2
$k \equiv 1835 \pmod{2312}$	1
$k \equiv 101 \pmod{2312}$	3
$k \equiv 679 \pmod{2312}$	4

congruence	p
$k \equiv 1699 \pmod{2312}$	5
$k \equiv 696 \pmod{1156}$	1
$k \equiv 985 \pmod{1156}$	2
$k \equiv 118 \pmod{1156}$	3
$k \equiv 407 \pmod{1156}$	4
$k \equiv 1121 \pmod{1156}$	5
$k \equiv 424 \pmod{867}$	1
$k \equiv 713 \pmod{867}$	2
$k \equiv 730 \pmod{867}$	3
$k \equiv 152 \pmod{867}$	4
$k \equiv 169 \pmod{867}$	6
$k \equiv 458 \pmod{867}$	5
$k \equiv 475 \pmod{867}$	7
$k \equiv 764 \pmod{867}$	8
$k \equiv 135 \pmod{1734}$	1
$k \equiv 441 \pmod{1734}$	2
$k \equiv 747 \pmod{1734}$	4
$k \equiv 1053 \pmod{1734}$	3
$k \equiv 1359 \pmod{1734}$	5
$k \equiv 1648 \pmod{1734}$	6
$k \equiv 1648 \pmod{2601}$	1
$k \equiv 2515 \pmod{2601}$	2
$k \equiv 781 \pmod{2601}$	3
$k \equiv 1937 \pmod{2601}$	4
$k \equiv 2804 \pmod{5202}$	1
$k \equiv 203 \pmod{5202}$	2
$k \equiv 1070 \pmod{5202}$	3
$k \equiv 3671 \pmod{5202}$	4
$k \equiv 220 \pmod{1445}$	3
$k \equiv 1376 \pmod{1445}$	1
$k \equiv 1087 \pmod{1445}$	2
$k \equiv 798 \pmod{1445}$	4
$k \equiv 509 \pmod{1445}$	5
$k \equiv 815 \pmod{1445}$	6
$k \equiv 1971 \pmod{2890}$	1
$k \equiv 526 \pmod{2890}$	2
$k \equiv 1682 \pmod{2890}$	3
$k \equiv 3127 \pmod{5780}$	9
$k \equiv 237 \pmod{5780}$	10
$k \equiv 5728 \pmod{5780}$	1
$k \equiv 1393 \pmod{5780}$	2
$k \equiv 2838 \pmod{5780}$	3
$k \equiv 4283 \pmod{5780}$	4
$k \equiv 1104 \pmod{5780}$	6
$k \equiv 2549 \pmod{5780}$	5
$k \equiv 3994 \pmod{5780}$	7
$k \equiv 5439 \pmod{5780}$	8

congruence	p
$k \equiv 543 \pmod{6936}$	1
$k \equiv 5167 \pmod{6936}$	2
$k \equiv 2855 \pmod{6936}$	3
$k \equiv 832 \pmod{3468}$	1
$k \equiv 2566 \pmod{3468}$	2
$k \equiv 1988 \pmod{3468}$	3
$k \equiv 254 \pmod{3468}$	4
$k \equiv 560 \pmod{2023}$	1
$k \equiv 1716 \pmod{2023}$	2
$k \equiv 849 \pmod{2023}$	3
$k \equiv 1138 \pmod{4046}$	1
$k \equiv 3161 \pmod{4046}$	2
$k \equiv 3450 \pmod{4046}$	3
$k \equiv 5473 \pmod{16184}$	1
$k \equiv 1427 \pmod{16184}$	2
$k \equiv 13565 \pmod{16184}$	3
$k \equiv 9519 \pmod{16184}$	4
$k \equiv 6340 \pmod{8092}$	1
$k \equiv 4317 \pmod{8092}$	2
$k \equiv 2294 \pmod{8092}$	3
$k \equiv 2005 \pmod{6069}$	1
$k \equiv 4028 \pmod{6069}$	2
$k \equiv 1155 \pmod{3179}$	1
$k \equiv 2311 \pmod{3179}$	2
$k \equiv 288 \pmod{3179}$	3
$k \equiv 1444 \pmod{3179}$	4
$k \equiv 2600 \pmod{3179}$	5
$k \equiv 6357 \pmod{6358}$	1
$k \equiv 3756 \pmod{6358}$	2
$k \equiv 577 \pmod{6358}$	3
$k \equiv 4912 \pmod{12716}$	1
$k \equiv 1733 \pmod{12716}$	2
$k \equiv 11270 \pmod{12716}$	3
$k \equiv 8091 \pmod{12716}$	4
$k \equiv 6068 \pmod{12716}$	5
$k \equiv 2889 \pmod{25432}$	2
$k \equiv 12426 \pmod{25432}$	1
$k \equiv 21963 \pmod{25432}$	3
$k \equiv 15605 \pmod{25432}$	4
$k \equiv 25142 \pmod{25432}$	5
$k \equiv 9247 \pmod{25432}$	6
$k \equiv 4045 \pmod{9537}$	1
$k \equiv 866 \pmod{9537}$	3
$k \equiv 8380 \pmod{9537}$	2
$k \equiv 5201 \pmod{9537}$	4
$k \equiv 16761 \pmod{19074}$	1
$k \equiv 11559 \pmod{19074}$	2

Table 7: Covering information for $d = -6$ (Part I)

congruence	p
$k \equiv 3 \pmod{6}$	1
$k \equiv 89 \pmod{90}$	2
$k \equiv 6 \pmod{16}$	1
$k \equiv 14 \pmod{28}$	1
$k \equiv 9 \pmod{32}$	1
$k \equiv 17 \pmod{18}$	1
$k \equiv 6 \pmod{22}$	1
$k \equiv 4 \pmod{44}$	1
$k \equiv 98 \pmod{99}$	1
$k \equiv 22 \pmod{30}$	1
$k \equiv 2 \pmod{30}$	2
$k \equiv 12 \pmod{30}$	3
$k \equiv 7 \pmod{10}$	1
$k \equiv 13 \pmod{20}$	1
$k \equiv 3 \pmod{20}$	2
$k \equiv 1 \pmod{25}$	1
$k \equiv 6 \pmod{25}$	2
$k \equiv 11 \pmod{25}$	3
$k \equiv 66 \pmod{75}$	1
$k \equiv 16 \pmod{75}$	2
$k \equiv 41 \pmod{75}$	3
$k \equiv 21 \pmod{125}$	1
$k \equiv 46 \pmod{125}$	2
$k \equiv 71 \pmod{125}$	3
$k \equiv 96 \pmod{125}$	4
$k \equiv 246 \pmod{250}$	1
$k \equiv 121 \pmod{250}$	2
$k \equiv 25 \pmod{50}$	1
$k \equiv 5 \pmod{50}$	2
$k \equiv 35 \pmod{50}$	3
$k \equiv 65 \pmod{100}$	1
$k \equiv 15 \pmod{100}$	2
$k \equiv 45 \pmod{100}$	3
$k \equiv 95 \pmod{100}$	4
$k \equiv 149 \pmod{180}$	1
$k \equiv 59 \pmod{180}$	2
$k \equiv 29 \pmod{180}$	3
$k \equiv 659 \pmod{720}$	1
$k \equiv 299 \pmod{720}$	2
$k \equiv 839 \pmod{1440}$	1
$k \equiv 1199 \pmod{1440}$	2
$k \equiv 119 \pmod{1440}$	3
$k \equiv 479 \pmod{1440}$	4
$k \equiv 19 \pmod{360}$	1
$k \equiv 139 \pmod{360}$	2
$k \equiv 259 \pmod{360}$	3
$k \equiv 679 \pmod{840}$	1
$k \equiv 799 \pmod{840}$	2
$k \equiv 79 \pmod{840}$	3
$k \equiv 199 \pmod{840}$	4
$k \equiv 319 \pmod{840}$	5
$k \equiv 439 \pmod{1680}$	1
$k \equiv 1279 \pmod{1680}$	2
$k \equiv 1399 \pmod{1680}$	3
$k \equiv 559 \pmod{1680}$	4
$k \equiv 49 \pmod{280}$	1
$k \equiv 189 \pmod{280}$	2
$k \equiv 64 \pmod{315}$	1
$k \equiv 274 \pmod{315}$	2
$k \equiv 169 \pmod{315}$	3
$k \equiv 289 \pmod{630}$	1
$k \equiv 499 \pmod{630}$	2
$k \equiv 79 \pmod{630}$	3
$k \equiv 199 \pmod{630}$	4
$k \equiv 409 \pmod{630}$	5
$k \equiv 1249 \pmod{1260}$	1
$k \equiv 109 \pmod{1260}$	2
$k \equiv 949 \pmod{1260}$	3
$k \equiv 529 \pmod{1260}$	4
$k \equiv 649 \pmod{1260}$	5
$k \equiv 229 \pmod{1260}$	6
$k \equiv 1069 \pmod{1260}$	7
$k \equiv 104 \pmod{175}$	1
$k \equiv 34 \pmod{175}$	2
$k \equiv 139 \pmod{175}$	3
$k \equiv 69 \pmod{350}$	1
$k \equiv 349 \pmod{350}$	2
$k \equiv 8 \pmod{40}$	1
$k \equiv 28 \pmod{40}$	2
$k \equiv 0 \pmod{29}$	1
$k \equiv 1 \pmod{29}$	2
$k \equiv 2 \pmod{29}$	3
$k \equiv 3 \pmod{29}$	4
$k \equiv 4 \pmod{29}$	5
$k \equiv 34 \pmod{58}$	1
$k \equiv 6 \pmod{58}$	2
$k \equiv 36 \pmod{116}$	1
$k \equiv 8 \pmod{116}$	2
$k \equiv 96 \pmod{116}$	3
$k \equiv 68 \pmod{116}$	4
$k \equiv 210 \pmod{232}$	1
$k \equiv 66 \pmod{232}$	2
$k \equiv 154 \pmod{232}$	3
$k \equiv 10 \pmod{232}$	4
$k \equiv 40 \pmod{232}$	5
$k \equiv 94 \pmod{464}$	1
$k \equiv 414 \pmod{464}$	2
$k \equiv 270 \pmod{464}$	3
$k \equiv 126 \pmod{464}$	4
$k \equiv 98 \pmod{464}$	5
$k \equiv 330 \pmod{464}$	6
$k \equiv 446 \pmod{464}$	7
$k \equiv 388 \pmod{928}$	1
$k \equiv 620 \pmod{928}$	2
$k \equiv 852 \pmod{928}$	3
$k \equiv 156 \pmod{928}$	4
$k \equiv 12 \pmod{87}$	1
$k \equiv 70 \pmod{87}$	2
$k \equiv 41 \pmod{87}$	3
$k \equiv 42 \pmod{174}$	1
$k \equiv 100 \pmod{174}$	2
$k \equiv 158 \pmod{174}$	3
$k \equiv 72 \pmod{348}$	1
$k \equiv 304 \pmod{348}$	2
$k \equiv 188 \pmod{348}$	3
$k \equiv 16 \pmod{348}$	4
$k \equiv 190 \pmod{348}$	5
$k \equiv 594 \pmod{696}$	1
$k \equiv 130 \pmod{696}$	2
$k \equiv 362 \pmod{696}$	3
$k \equiv 942 \pmod{1392}$	1
$k \equiv 478 \pmod{1392}$	2
$k \equiv 14 \pmod{1392}$	3
$k \equiv 189 \pmod{261}$	1
$k \equiv 73 \pmod{261}$	2
$k \equiv 218 \pmod{261}$	3
$k \equiv 102 \pmod{261}$	4
$k \equiv 247 \pmod{261}$	5
$k \equiv 131 \pmod{261}$	6
$k \equiv 15 \pmod{261}$	7
$k \equiv 160 \pmod{261}$	8
$k \equiv 44 \pmod{522}$	1
$k \equiv 306 \pmod{522}$	2
$k \equiv 480 \pmod{522}$	3
$k \equiv 132 \pmod{522}$	4
$k \equiv 596 \pmod{1044}$	1
$k \equiv 74 \pmod{1044}$	2
$k \equiv 248 \pmod{1044}$	4
$k \equiv 770 \pmod{1044}$	5
$k \equiv 944 \pmod{1044}$	3
$k \equiv 422 \pmod{1044}$	6
$k \equiv 48 \pmod{60}$	1
$k \equiv 28 \pmod{60}$	2
$k \equiv 8 \pmod{60}$	3
$k \equiv 75 \pmod{145}$	1
$k \equiv 104 \pmod{145}$	2
$k \equiv 105 \pmod{145}$	3
$k \equiv 134 \pmod{145}$	4
$k \equiv 280 \pmod{290}$	1
$k \equiv 164 \pmod{290}$	2

Table 7: Covering information for $d = -6$ (Part II)

congruence	p
$k \equiv 20 \pmod{290}$	3
$k \equiv 194 \pmod{290}$	4
$k \equiv 50 \pmod{290}$	5
$k \equiv 224 \pmod{290}$	6
$k \equiv 278 \pmod{580}$	1
$k \equiv 18 \pmod{580}$	2
$k \equiv 338 \pmod{580}$	3
$k \equiv 78 \pmod{580}$	4
$k \equiv 978 \pmod{1160}$	1
$k \equiv 398 \pmod{1160}$	2
$k \equiv 225 \pmod{435}$	1
$k \equiv 370 \pmod{435}$	2
$k \equiv 80 \pmod{435}$	3
$k \equiv 399 \pmod{435}$	4
$k \equiv 544 \pmod{870}$	1
$k \equiv 254 \pmod{870}$	2
$k \equiv 690 \pmod{870}$	3
$k \equiv 400 \pmod{870}$	4
$k \equiv 110 \pmod{870}$	5
$k \equiv 864 \pmod{870}$	6
$k \equiv 574 \pmod{870}$	7
$k \equiv 2024 \pmod{3480}$	1
$k \equiv 1154 \pmod{3480}$	2
$k \equiv 284 \pmod{3480}$	3
$k \equiv 6374 \pmod{6960}$	1
$k \equiv 2894 \pmod{6960}$	2
$k \equiv 138 \pmod{1740}$	1
$k \equiv 718 \pmod{1740}$	2
$k \equiv 1298 \pmod{1740}$	3
$k \equiv 1038 \pmod{1740}$	4
$k \equiv 1618 \pmod{1740}$	5
$k \equiv 458 \pmod{1740}$	6
$k \equiv 198 \pmod{1740}$	7
$k \equiv 778 \pmod{1740}$	8
$k \equiv 1358 \pmod{1740}$	9
$k \equiv 140 \pmod{1015}$	1

congruence	p
$k \equiv 575 \pmod{1015}$	2
$k \equiv 1010 \pmod{1015}$	3
$k \equiv 430 \pmod{1015}$	4
$k \equiv 865 \pmod{1015}$	5
$k \equiv 285 \pmod{1015}$	6
$k \equiv 720 \pmod{1015}$	7
$k \equiv 1764 \pmod{2030}$	1
$k \equiv 1184 \pmod{2030}$	2
$k \equiv 604 \pmod{2030}$	3
$k \equiv 24 \pmod{2030}$	4
$k \equiv 3504 \pmod{4060}$	1
$k \equiv 1474 \pmod{4060}$	2
$k \equiv 2924 \pmod{4060}$	3
$k \equiv 894 \pmod{4060}$	4
$k \equiv 2344 \pmod{4060}$	5
$k \equiv 314 \pmod{4060}$	6
$k \equiv 141 \pmod{203}$	1
$k \equiv 170 \pmod{203}$	2
$k \equiv 199 \pmod{203}$	3
$k \equiv 25 \pmod{203}$	4
$k \equiv 54 \pmod{406}$	1
$k \equiv 286 \pmod{406}$	2
$k \equiv 316 \pmod{406}$	3
$k \equiv 142 \pmod{406}$	4
$k \equiv 374 \pmod{406}$	5
$k \equiv 112 \pmod{812}$	1
$k \equiv 84 \pmod{812}$	2
$k \equiv 200 \pmod{812}$	3
$k \equiv 606 \pmod{812}$	4
$k \equiv 432 \pmod{812}$	5
$k \equiv 26 \pmod{812}$	6
$k \equiv 258 \pmod{609}$	1
$k \equiv 55 \pmod{609}$	2
$k \equiv 461 \pmod{609}$	3
$k \equiv 143 \pmod{319}$	1
$k \equiv 56 \pmod{319}$	2

congruence	p
$k \equiv 288 \pmod{319}$	3
$k \equiv 201 \pmod{319}$	4
$k \equiv 346 \pmod{638}$	1
$k \equiv 578 \pmod{638}$	2
$k \equiv 172 \pmod{638}$	3
$k \equiv 404 \pmod{638}$	4
$k \equiv 636 \pmod{638}$	5
$k \equiv 114 \pmod{1276}$	1
$k \equiv 868 \pmod{1276}$	2
$k \equiv 230 \pmod{1276}$	3
$k \equiv 260 \pmod{377}$	1
$k \equiv 144 \pmod{377}$	2
$k \equiv 28 \pmod{377}$	3
$k \equiv 289 \pmod{377}$	4
$k \equiv 550 \pmod{754}$	1
$k \equiv 434 \pmod{754}$	2
$k \equiv 1072 \pmod{1508}$	3
$k \equiv 318 \pmod{1508}$	1
$k \equiv 956 \pmod{1508}$	2
$k \equiv 202 \pmod{1508}$	4
$k \equiv 840 \pmod{1131}$	1
$k \equiv 463 \pmod{1131}$	2
$k \equiv 86 \pmod{1131}$	3
$k \equiv 1101 \pmod{1131}$	4
$k \equiv 724 \pmod{2262}$	1
$k \equiv 1478 \pmod{2262}$	2
$k \equiv 985 \pmod{1885}$	1
$k \equiv 608 \pmod{1885}$	2
$k \equiv 1739 \pmod{1885}$	3
$k \equiv 2000 \pmod{3770}$	1
$k \equiv 3508 \pmod{3770}$	3
$k \equiv 2754 \pmod{3770}$	2
$k \equiv 1130 \pmod{3770}$	4
$k \equiv 2638 \pmod{3770}$	5
$k \equiv 1884 \pmod{3770}$	6

Table 8: Covering information for $d = -5$ (Part I)

congruence	p
$k \equiv 0 \pmod{13}$	1
$k \equiv 1 \pmod{13}$	2
$k \equiv 2 \pmod{13}$	3
$k \equiv 16 \pmod{26}$	1
$k \equiv 3 \pmod{26}$	2
$k \equiv 4 \pmod{52}$	1
$k \equiv 17 \pmod{52}$	2
$k \equiv 82 \pmod{208}$	1
$k \equiv 134 \pmod{208}$	2

congruence	p
$k \equiv 186 \pmod{208}$	3
$k \equiv 30 \pmod{208}$	4
$k \equiv 147 \pmod{208}$	5
$k \equiv 199 \pmod{208}$	6
$k \equiv 43 \pmod{416}$	1
$k \equiv 251 \pmod{416}$	2
$k \equiv 303 \pmod{416}$	3
$k \equiv 95 \pmod{416}$	4
$k \equiv 18 \pmod{39}$	1

congruence	p
$k \equiv 70 \pmod{78}$	1
$k \equiv 31 \pmod{78}$	2
$k \equiv 44 \pmod{78}$	3
$k \equiv 5 \pmod{78}$	4
$k \equiv 1 \pmod{6}$	1
$k \equiv 45 \pmod{117}$	1
$k \equiv 84 \pmod{117}$	2
$k \equiv 6 \pmod{117}$	3
$k \equiv 58 \pmod{234}$	1

Table 8: Covering information for $d = -5$ (Part II)

congruence	p
$k \equiv 136 \pmod{234}$	2
$k \equiv 214 \pmod{234}$	3
$k \equiv 110 \pmod{117}$	4
$k \equiv 32 \pmod{351}$	1
$k \equiv 149 \pmod{351}$	2
$k \equiv 266 \pmod{351}$	3
$k \equiv 305 \pmod{351}$	4
$k \equiv 71 \pmod{702}$	1
$k \equiv 188 \pmod{702}$	2
$k \equiv 422 \pmod{702}$	3
$k \equiv 539 \pmod{702}$	4
$k \equiv 20 \pmod{65}$	1
$k \equiv 46 \pmod{65}$	2
$k \equiv 7 \pmod{130}$	1
$k \equiv 72 \pmod{130}$	2
$k \equiv 33 \pmod{390}$	1
$k \equiv 163 \pmod{390}$	2
$k \equiv 293 \pmod{390}$	3
$k \equiv 228 \pmod{520}$	1
$k \equiv 488 \pmod{520}$	2
$k \equiv 98 \pmod{1040}$	1
$k \equiv 358 \pmod{1040}$	2
$k \equiv 618 \pmod{1040}$	3
$k \equiv 878 \pmod{1040}$	4
$k \equiv 59 \pmod{195}$	1
$k \equiv 124 \pmod{195}$	2
$k \equiv 189 \pmod{195}$	3
$k \equiv 8 \pmod{104}$	1
$k \equiv 60 \pmod{104}$	2
$k \equiv 21 \pmod{156}$	1
$k \equiv 73 \pmod{156}$	2
$k \equiv 125 \pmod{156}$	3
$k \equiv 34 \pmod{312}$	1
$k \equiv 86 \pmod{312}$	2
$k \equiv 138 \pmod{312}$	3
$k \equiv 242 \pmod{312}$	4
$k \equiv 294 \pmod{312}$	5
$k \equiv 190 \pmod{624}$	1
$k \equiv 502 \pmod{624}$	2
$k \equiv 47 \pmod{260}$	1
$k \equiv 99 \pmod{260}$	2
$k \equiv 151 \pmod{260}$	3
$k \equiv 203 \pmod{260}$	4
$k \equiv 255 \pmod{260}$	5
$k \equiv 9 \pmod{91}$	1
$k \equiv 22 \pmod{91}$	2
$k \equiv 35 \pmod{91}$	3
$k \equiv 48 \pmod{91}$	4
$k \equiv 61 \pmod{91}$	5
$k \equiv 74 \pmod{91}$	6

congruence	p
$k \equiv 87 \pmod{91}$	7
$k \equiv 49 \pmod{182}$	1
$k \equiv 140 \pmod{182}$	2
$k \equiv 36 \pmod{364}$	1
$k \equiv 127 \pmod{364}$	2
$k \equiv 309 \pmod{364}$	3
$k \equiv 218 \pmod{728}$	1
$k \equiv 582 \pmod{728}$	2
$k \equiv 23 \pmod{273}$	1
$k \equiv 114 \pmod{273}$	2
$k \equiv 205 \pmod{273}$	3
$k \equiv 10 \pmod{273}$	4
$k \equiv 192 \pmod{273}$	5
$k \equiv 101 \pmod{546}$	1
$k \equiv 374 \pmod{546}$	2
$k \equiv 88 \pmod{455}$	1
$k \equiv 179 \pmod{455}$	2
$k \equiv 270 \pmod{455}$	3
$k \equiv 361 \pmod{455}$	4
$k \equiv 452 \pmod{455}$	5
$k \equiv 75 \pmod{910}$	1
$k \equiv 257 \pmod{910}$	2
$k \equiv 439 \pmod{910}$	3
$k \equiv 621 \pmod{910}$	4
$k \equiv 803 \pmod{910}$	5
$k \equiv 348 \pmod{1456}$	1
$k \equiv 530 \pmod{1456}$	2
$k \equiv 712 \pmod{1456}$	3
$k \equiv 1076 \pmod{1456}$	4
$k \equiv 1258 \pmod{1456}$	5
$k \equiv 1440 \pmod{1456}$	6
$k \equiv 166 \pmod{2912}$	1
$k \equiv 1622 \pmod{2912}$	2
$k \equiv 2350 \pmod{2912}$	3
$k \equiv 62 \pmod{416}$	5
$k \equiv 244 \pmod{1001}$	1
$k \equiv 517 \pmod{1001}$	2
$k \equiv 608 \pmod{1001}$	3
$k \equiv 881 \pmod{1001}$	4
$k \equiv 335 \pmod{2002}$	1
$k \equiv 972 \pmod{2002}$	2
$k \equiv 1336 \pmod{2002}$	3
$k \equiv 1973 \pmod{2002}$	4
$k \equiv 1700 \pmod{2002}$	5
$k \equiv 699 \pmod{4004}$	1
$k \equiv 2701 \pmod{4004}$	2
$k \equiv 62 \pmod{3003}$	1
$k \equiv 1063 \pmod{3003}$	2
$k \equiv 2064 \pmod{3003}$	3
$k \equiv 426 \pmod{6006}$	1

congruence	p
$k \equiv 1427 \pmod{6006}$	2
$k \equiv 2428 \pmod{6006}$	3
$k \equiv 3429 \pmod{6006}$	4
$k \equiv 4430 \pmod{6006}$	5
$k \equiv 5431 \pmod{6006}$	6
$k \equiv 790 \pmod{5005}$	1
$k \equiv 1791 \pmod{5005}$	2
$k \equiv 2792 \pmod{5005}$	3
$k \equiv 3793 \pmod{15015}$	1
$k \equiv 8798 \pmod{15015}$	2
$k \equiv 13803 \pmod{15015}$	3
$k \equiv 4794 \pmod{30030}$	1
$k \equiv 9799 \pmod{30030}$	2
$k \equiv 14804 \pmod{30030}$	3
$k \equiv 19809 \pmod{30030}$	5
$k \equiv 24814 \pmod{30030}$	4
$k \equiv 29819 \pmod{30030}$	6
$k \equiv 1154 \pmod{8008}$	2
$k \equiv 3156 \pmod{8008}$	1
$k \equiv 5158 \pmod{8008}$	3
$k \equiv 7160 \pmod{8008}$	4
$k \equiv 153 \pmod{10010}$	1
$k \equiv 2155 \pmod{10010}$	2
$k \equiv 4157 \pmod{10010}$	3
$k \equiv 8161 \pmod{10010}$	4
$k \equiv 6159 \pmod{60060}$	1
$k \equiv 16169 \pmod{60060}$	2
$k \equiv 26179 \pmod{60060}$	3
$k \equiv 36189 \pmod{60060}$	4
$k \equiv 46199 \pmod{60060}$	5
$k \equiv 56209 \pmod{60060}$	6
$k \equiv 11 \pmod{143}$	1
$k \equiv 24 \pmod{143}$	2
$k \equiv 89 \pmod{143}$	3
$k \equiv 102 \pmod{143}$	4
$k \equiv 37 \pmod{286}$	1
$k \equiv 115 \pmod{286}$	2
$k \equiv 180 \pmod{286}$	3
$k \equiv 258 \pmod{286}$	4
$k \equiv 50 \pmod{429}$	1
$k \equiv 193 \pmod{429}$	2
$k \equiv 336 \pmod{429}$	3
$k \equiv 128 \pmod{572}$	1
$k \equiv 271 \pmod{572}$	2
$k \equiv 414 \pmod{572}$	3
$k \equiv 557 \pmod{572}$	4
$k \equiv 63 \pmod{1716}$	1
$k \equiv 635 \pmod{1716}$	2
$k \equiv 1207 \pmod{1716}$	3
$k \equiv 349 \pmod{572}$	5

Table 8: Covering information for $d = -5$ (Part III)

congruence	p
$k \equiv 206 \pmod{286}$	5
$k \equiv 141 \pmod{858}$	1
$k \equiv 284 \pmod{858}$	2
$k \equiv 427 \pmod{858}$	3
$k \equiv 570 \pmod{858}$	4
$k \equiv 856 \pmod{858}$	5
$k \equiv 713 \pmod{1716}$	4
$k \equiv 1571 \pmod{3432}$	1
$k \equiv 3287 \pmod{3432}$	2
$k \equiv 76 \pmod{715}$	1
$k \equiv 362 \pmod{715}$	2
$k \equiv 505 \pmod{715}$	3
$k \equiv 219 \pmod{1430}$	1
$k \equiv 648 \pmod{1430}$	2
$k \equiv 934 \pmod{1430}$	3
$k \equiv 1363 \pmod{1430}$	4
$k \equiv 12 \pmod{169}$	1
$k \equiv 25 \pmod{169}$	2
$k \equiv 38 \pmod{169}$	3
$k \equiv 51 \pmod{338}$	1
$k \equiv 64 \pmod{338}$	2
$k \equiv 220 \pmod{338}$	3
$k \equiv 233 \pmod{338}$	4
$k \equiv 77 \pmod{507}$	2
$k \equiv 90 \pmod{507}$	1
$k \equiv 246 \pmod{507}$	3
$k \equiv 259 \pmod{507}$	4
$k \equiv 415 \pmod{507}$	5
$k \equiv 428 \pmod{507}$	6
$k \equiv 441 \pmod{507}$	7
$k \equiv 103 \pmod{1014}$	1

congruence	p
$k \equiv 272 \pmod{1014}$	2
$k \equiv 610 \pmod{1014}$	3
$k \equiv 779 \pmod{1014}$	4
$k \equiv 116 \pmod{676}$	1
$k \equiv 285 \pmod{676}$	2
$k \equiv 454 \pmod{676}$	3
$k \equiv 623 \pmod{2704}$	1
$k \equiv 1299 \pmod{2704}$	2
$k \equiv 1975 \pmod{2704}$	3
$k \equiv 2651 \pmod{2704}$	4
$k \equiv 129 \pmod{1352}$	1
$k \equiv 298 \pmod{1352}$	2
$k \equiv 467 \pmod{1352}$	3
$k \equiv 636 \pmod{1352}$	4
$k \equiv 805 \pmod{1352}$	5
$k \equiv 974 \pmod{1352}$	6
$k \equiv 1143 \pmod{1352}$	7
$k \equiv 1312 \pmod{1352}$	8
$k \equiv 311 \pmod{845}$	1
$k \equiv 480 \pmod{845}$	2
$k \equiv 142 \pmod{1690}$	1
$k \equiv 818 \pmod{1690}$	2
$k \equiv 987 \pmod{1690}$	3
$k \equiv 1663 \pmod{1690}$	4
$k \equiv 649 \pmod{3380}$	1
$k \equiv 1494 \pmod{3380}$	2
$k \equiv 2339 \pmod{3380}$	3
$k \equiv 3184 \pmod{3380}$	4
$k \equiv 155 \pmod{1183}$	1
$k \equiv 324 \pmod{1183}$	2
$k \equiv 493 \pmod{1183}$	3

congruence	p
$k \equiv 1169 \pmod{1183}$	4
$k \equiv 662 \pmod{2366}$	1
$k \equiv 831 \pmod{2366}$	2
$k \equiv 1845 \pmod{2366}$	3
$k \equiv 2014 \pmod{2366}$	4
$k \equiv 1000 \pmod{3549}$	1
$k \equiv 2183 \pmod{3549}$	2
$k \equiv 3366 \pmod{3549}$	3
$k \equiv 168 \pmod{2535}$	1
$k \equiv 675 \pmod{2535}$	2
$k \equiv 1182 \pmod{2535}$	3
$k \equiv 2196 \pmod{2535}$	4
$k \equiv 1689 \pmod{5070}$	1
$k \equiv 4224 \pmod{5070}$	2
$k \equiv 506 \pmod{1521}$	1
$k \equiv 1013 \pmod{3042}$	1
$k \equiv 1520 \pmod{3042}$	2
$k \equiv 2534 \pmod{3042}$	3
$k \equiv 3041 \pmod{3042}$	4
$k \equiv 844 \pmod{10140}$	1
$k \equiv 1858 \pmod{10140}$	2
$k \equiv 2872 \pmod{10140}$	3
$k \equiv 3886 \pmod{10140}$	4
$k \equiv 4900 \pmod{10140}$	8
$k \equiv 5914 \pmod{10140}$	5
$k \equiv 6928 \pmod{10140}$	6
$k \equiv 7942 \pmod{10140}$	7
$k \equiv 8956 \pmod{10140}$	9
$k \equiv 9970 \pmod{10140}$	10

Table 9: Covering information for $d = -3$ (Part I)

congruence	p
$k \equiv 4 \pmod{6}$	2
$k \equiv 5 \pmod{6}$	1
$k \equiv 0 \pmod{16}$	1
$k \equiv 11 \pmod{21}$	1
$k \equiv 14 \pmod{22}$	1
$k \equiv 40 \pmod{44}$	1
$k \equiv 96 \pmod{99}$	1
$k \equiv 8 \pmod{64}$	1
$k \equiv 24 \pmod{64}$	2
$k \equiv 40 \pmod{64}$	3
$k \equiv 56 \pmod{64}$	4
$k \equiv 25 \pmod{210}$	1
$k \equiv 151 \pmod{210}$	2
$k \equiv 67 \pmod{210}$	3

congruence	p
$k \equiv 193 \pmod{420}$	1
$k \equiv 403 \pmod{420}$	2
$k \equiv 109 \pmod{420}$	3
$k \equiv 319 \pmod{420}$	4
$k \equiv 18 \pmod{336}$	1
$k \equiv 228 \pmod{336}$	2
$k \equiv 102 \pmod{336}$	3
$k \equiv 522 \pmod{672}$	1
$k \equiv 186 \pmod{672}$	2
$k \equiv 396 \pmod{672}$	3
$k \equiv 60 \pmod{672}$	4
$k \equiv 270 \pmod{672}$	5
$k \equiv 606 \pmod{672}$	6
$k \equiv 81 \pmod{126}$	1

congruence	p
$k \equiv 39 \pmod{126}$	2
$k \equiv 249 \pmod{252}$	1
$k \equiv 123 \pmod{252}$	2
$k \equiv 113 \pmod{176}$	1
$k \equiv 69 \pmod{176}$	2
$k \equiv 201 \pmod{352}$	1
$k \equiv 333 \pmod{352}$	2
$k \equiv 25 \pmod{352}$	3
$k \equiv 157 \pmod{352}$	4
$k \equiv 3 \pmod{704}$	1
$k \equiv 531 \pmod{704}$	2
$k \equiv 355 \pmod{704}$	3
$k \equiv 179 \pmod{704}$	4
$k \equiv 267 \pmod{1056}$	1

Table 9: Covering information for $d = -3$ (Part II)

congruence	p
$k \equiv 619 \pmod{1056}$	2
$k \equiv 971 \pmod{1056}$	3
$k \equiv 795 \pmod{2112}$	1
$k \equiv 91 \pmod{2112}$	2
$k \equiv 1499 \pmod{2112}$	3
$k \equiv 1851 \pmod{2112}$	4
$k \equiv 1147 \pmod{2112}$	5
$k \equiv 443 \pmod{4224}$	1
$k \equiv 2555 \pmod{4224}$	2
$k \equiv 135 \pmod{440}$	1
$k \equiv 311 \pmod{440}$	2
$k \equiv 487 \pmod{880}$	1
$k \equiv 47 \pmod{880}$	2
$k \equiv 663 \pmod{880}$	3
$k \equiv 223 \pmod{880}$	4
$k \equiv 839 \pmod{880}$	5
$k \equiv 399 \pmod{1760}$	1
$k \equiv 1279 \pmod{1760}$	2
$k \equiv 0 \pmod{231}$	1
$k \equiv 99 \pmod{231}$	2
$k \equiv 198 \pmod{231}$	3
$k \equiv 66 \pmod{231}$	4
$k \equiv 33 \pmod{231}$	5
$k \equiv 132 \pmod{231}$	6
$k \equiv 385 \pmod{462}$	1
$k \equiv 253 \pmod{462}$	2
$k \equiv 121 \pmod{462}$	3
$k \equiv 770 \pmod{924}$	1
$k \equiv 638 \pmod{924}$	2
$k \equiv 506 \pmod{924}$	3
$k \equiv 374 \pmod{924}$	4
$k \equiv 110 \pmod{924}$	5
$k \equiv 308 \pmod{1848}$	1
$k \equiv 1100 \pmod{1848}$	2
$k \equiv 44 \pmod{1848}$	3
$k \equiv 836 \pmod{1848}$	4
$k \equiv 451 \pmod{693}$	1
$k \equiv 220 \pmod{693}$	2
$k \equiv 682 \pmod{693}$	3
$k \equiv 649 \pmod{693}$	4
$k \equiv 418 \pmod{693}$	5
$k \equiv 187 \pmod{693}$	6
$k \equiv 55 \pmod{1386}$	1
$k \equiv 979 \pmod{1386}$	2
$k \equiv 517 \pmod{1386}$	3
$k \equiv 1826 \pmod{3696}$	1
$k \equiv 2750 \pmod{3696}$	2
$k \equiv 3674 \pmod{3696}$	3
$k \equiv 902 \pmod{3696}$	4
$k \equiv 2420 \pmod{7392}$	1

congruence	p
$k \equiv 4268 \pmod{7392}$	2
$k \equiv 6116 \pmod{7392}$	3
$k \equiv 572 \pmod{7392}$	4
$k \equiv 5060 \pmod{5544}$	1
$k \equiv 3212 \pmod{5544}$	2
$k \equiv 1364 \pmod{5544}$	3
$k \equiv 0 \pmod{19}$	1
$k \equiv 1 \pmod{38}$	1
$k \equiv 58 \pmod{76}$	1
$k \equiv 58 \pmod{76}$	2
$k \equiv 20 \pmod{152}$	1
$k \equiv 116 \pmod{152}$	2
$k \equiv 2 \pmod{152}$	3
$k \equiv 78 \pmod{152}$	4
$k \equiv 97 \pmod{304}$	1
$k \equiv 211 \pmod{304}$	2
$k \equiv 21 \pmod{304}$	3
$k \equiv 135 \pmod{608}$	1
$k \equiv 439 \pmod{608}$	2
$k \equiv 553 \pmod{608}$	3
$k \equiv 857 \pmod{1216}$	1
$k \equiv 249 \pmod{1216}$	2
$k \equiv 971 \pmod{1216}$	3
$k \equiv 667 \pmod{1216}$	4
$k \equiv 363 \pmod{1216}$	5
$k \equiv 59 \pmod{1216}$	7
$k \equiv 781 \pmod{1216}$	6
$k \equiv 477 \pmod{1216}$	8
$k \equiv 173 \pmod{1216}$	9
$k \equiv 1085 \pmod{2432}$	1
$k \equiv 2301 \pmod{2432}$	2
$k \equiv 1807 \pmod{2432}$	3
$k \equiv 63 \pmod{192}$	4
$k \equiv 159 \pmod{384}$	1
$k \equiv 31 \pmod{384}$	2
$k \equiv 303 \pmod{384}$	3
$k \equiv 127 \pmod{384}$	4
$k \equiv 319 \pmod{384}$	5
$k \equiv 559 \pmod{768}$	1
$k \equiv 175 \pmod{768}$	2
$k \equiv 591 \pmod{768}$	3
$k \equiv 79 \pmod{768}$	4
$k \equiv 207 \pmod{768}$	5
$k \equiv 463 \pmod{768}$	6
$k \equiv 3935 \pmod{4864}$	1
$k \equiv 1503 \pmod{4864}$	2
$k \equiv 2415 \pmod{4864}$	3
$k \equiv 4847 \pmod{4864}$	4
$k \equiv 3 \pmod{57}$	1
$k \equiv 42 \pmod{57}$	2

congruence	p
$k \equiv 24 \pmod{57}$	3
$k \equiv 79 \pmod{114}$	1
$k \equiv 98 \pmod{114}$	2
$k \equiv 61 \pmod{228}$	1
$k \equiv 175 \pmod{228}$	2
$k \equiv 194 \pmod{228}$	3
$k \equiv 157 \pmod{228}$	4
$k \equiv 43 \pmod{228}$	5
$k \equiv 62 \pmod{228}$	6
$k \equiv 308 \pmod{456}$	1
$k \equiv 404 \pmod{456}$	2
$k \equiv 25 \pmod{456}$	3
$k \equiv 367 \pmod{456}$	4
$k \equiv 253 \pmod{456}$	5
$k \equiv 63 \pmod{171}$	1
$k \equiv 120 \pmod{171}$	2
$k \equiv 6 \pmod{171}$	3
$k \equiv 595 \pmod{912}$	1
$k \equiv 139 \pmod{912}$	2
$k \equiv 956 \pmod{1368}$	1
$k \equiv 44 \pmod{1368}$	2
$k \equiv 500 \pmod{1368}$	3
$k \equiv 1526 \pmod{5472}$	1
$k \equiv 3350 \pmod{5472}$	2
$k \equiv 5174 \pmod{5472}$	3
$k \equiv 45 \pmod{342}$	1
$k \equiv 216 \pmod{342}$	2
$k \equiv 273 \pmod{342}$	3
$k \equiv 102 \pmod{342}$	4
$k \equiv 159 \pmod{342}$	5
$k \equiv 330 \pmod{3420}$	1
$k \equiv 3066 \pmod{3420}$	2
$k \equiv 2382 \pmod{3420}$	3
$k \equiv 1698 \pmod{3420}$	4
$k \equiv 1014 \pmod{3420}$	5
$k \equiv 5460 \pmod{6840}$	1
$k \equiv 1356 \pmod{6840}$	2
$k \equiv 4092 \pmod{6840}$	3
$k \equiv 6828 \pmod{6840}$	4
$k \equiv 2724 \pmod{6840}$	5
$k \equiv 577 \pmod{684}$	1
$k \equiv 7 \pmod{684}$	2
$k \equiv 121 \pmod{684}$	3
$k \equiv 235 \pmod{684}$	4
$k \equiv 1033 \pmod{1368}$	4
$k \equiv 349 \pmod{2736}$	1
$k \equiv 1717 \pmod{2736}$	2
$k \equiv 463 \pmod{2736}$	3
$k \equiv 5251 \pmod{5472}$	4
$k \equiv 1831 \pmod{5472}$	5

Table 9: Covering information for $d = -3$ (Part III)

congruence	p	congruence	p	congruence	p
$k \equiv 3883 \pmod{5472}$	6	$k \equiv 10 \pmod{532}$	1	$k \equiv 885 \pmod{1197}$	2
$k \equiv 2515 \pmod{5472}$	7	$k \equiv 390 \pmod{532}$	2	$k \equiv 87 \pmod{1197}$	3
$k \equiv 10039 \pmod{10944}$	1	$k \equiv 314 \pmod{532}$	3	$k \equiv 2215 \pmod{2394}$	1
$k \equiv 6619 \pmod{10944}$	2	$k \equiv 49 \pmod{532}$	4	$k \equiv 1417 \pmod{2394}$	2
$k \equiv 4567 \pmod{10944}$	3	$k \equiv 182 \pmod{532}$	5	$k \equiv 619 \pmod{2394}$	3
$k \equiv 1147 \pmod{10944}$	4	$k \equiv 276 \pmod{1064}$	1	$k \equiv 1550 \pmod{2394}$	4
$k \equiv 140 \pmod{570}$	1	$k \equiv 124 \pmod{1064}$	2	$k \equiv 752 \pmod{2394}$	5
$k \equiv 26 \pmod{570}$	2	$k \equiv 580 \pmod{1064}$	3	$k \equiv 201 \pmod{931}$	1
$k \equiv 482 \pmod{570}$	3	$k \equiv 1379 \pmod{2128}$	1	$k \equiv 600 \pmod{931}$	2
$k \equiv 368 \pmod{570}$	4	$k \equiv 1911 \pmod{2128}$	2	$k \equiv 68 \pmod{931}$	3
$k \equiv 254 \pmod{1140}$	1	$k \equiv 315 \pmod{2128}$	3	$k \equiv 467 \pmod{931}$	4
$k \equiv 824 \pmod{1140}$	2	$k \equiv 847 \pmod{2128}$	4	$k \equiv 866 \pmod{931}$	5
$k \equiv 65 \pmod{95}$	1	$k \equiv 3108 \pmod{4256}$	1	$k \equiv 334 \pmod{931}$	6
$k \equiv 46 \pmod{95}$	2	$k \equiv 4172 \pmod{4256}$	2	$k \equiv 733 \pmod{931}$	7
$k \equiv 27 \pmod{95}$	3	$k \equiv 980 \pmod{4256}$	3	$k \equiv 790 \pmod{1862}$	1
$k \equiv 8 \pmod{95}$	4	$k \equiv 2044 \pmod{4256}$	4	$k \equiv 1721 \pmod{3724}$	1
$k \equiv 84 \pmod{95}$	5	$k \equiv 505 \pmod{665}$	1	$k \equiv 3583 \pmod{3724}$	2
$k \equiv 66 \pmod{285}$	1	$k \equiv 106 \pmod{665}$	2	$k \equiv 2120 \pmod{3724}$	3
$k \equiv 256 \pmod{285}$	2	$k \equiv 372 \pmod{665}$	3	$k \equiv 1189 \pmod{3724}$	4
$k \equiv 161 \pmod{285}$	3	$k \equiv 1303 \pmod{1330}$	1	$k \equiv 258 \pmod{3724}$	5
$k \equiv 142 \pmod{190}$	1	$k \equiv 239 \pmod{1330}$	2	$k \equiv 3051 \pmod{3724}$	6
$k \equiv 123 \pmod{190}$	2	$k \equiv 1094 \pmod{1330}$	3	$k \equiv 657 \pmod{7448}$	1
$k \equiv 104 \pmod{190}$	3	$k \equiv 638 \pmod{2660}$	1	$k \equiv 3450 \pmod{7448}$	2
$k \equiv 85 \pmod{380}$	1	$k \equiv 2234 \pmod{2660}$	2	$k \equiv 6243 \pmod{7448}$	3
$k \equiv 275 \pmod{380}$	2	$k \equiv 4628 \pmod{5320}$	1	$k \equiv 1588 \pmod{7448}$	5
$k \equiv 47 \pmod{380}$	3	$k \equiv 3564 \pmod{5320}$	2	$k \equiv 4381 \pmod{7448}$	4
$k \equiv 237 \pmod{380}$	4	$k \equiv 30 \pmod{1995}$	1	$k \equiv 7174 \pmod{7448}$	6
$k \equiv 218 \pmod{380}$	5	$k \equiv 1626 \pmod{1995}$	2	$k \equiv 2519 \pmod{7448}$	7
$k \equiv 9 \pmod{380}$	6	$k \equiv 1227 \pmod{1995}$	3	$k \equiv 1056 \pmod{2793}$	1
$k \equiv 199 \pmod{380}$	7	$k \equiv 828 \pmod{1995}$	4	$k \equiv 1455 \pmod{2793}$	3
$k \equiv 370 \pmod{380}$	8	$k \equiv 3355 \pmod{3990}$	1	$k \equiv 1854 \pmod{2793}$	2
$k \equiv 180 \pmod{760}$	1	$k \equiv 961 \pmod{3990}$	2	$k \equiv 2253 \pmod{2793}$	4
$k \equiv 28 \pmod{760}$	2	$k \equiv 2557 \pmod{7980}$	1	$k \equiv 1987 \pmod{5586}$	1
$k \equiv 145 \pmod{1140}$	3	$k \equiv 6547 \pmod{7980}$	2	$k \equiv 5179 \pmod{5586}$	2
$k \equiv 715 \pmod{1140}$	5	$k \equiv 4153 \pmod{7980}$	3	$k \equiv 2785 \pmod{5586}$	3
$k \equiv 601 \pmod{1140}$	6	$k \equiv 163 \pmod{7980}$	4	$k \equiv 391 \pmod{5586}$	4
$k \equiv 31 \pmod{1140}$	4	$k \equiv 2690 \pmod{7980}$	5	$k \equiv 2918 \pmod{11172}$	2
$k \equiv 1057 \pmod{1140}$	8	$k \equiv 4286 \pmod{7980}$	6	$k \equiv 6110 \pmod{11172}$	1
$k \equiv 487 \pmod{1140}$	7	$k \equiv 5882 \pmod{7980}$	8	$k \equiv 9302 \pmod{11172}$	3
$k \equiv 373 \pmod{1140}$	9	$k \equiv 7478 \pmod{7980}$	7	$k \equiv 1322 \pmod{11172}$	4
$k \equiv 943 \pmod{1140}$	10	$k \equiv 5749 \pmod{7980}$	9	$k \equiv 19676 \pmod{22344}$	2
$k \equiv 829 \pmod{1140}$	11	$k \equiv 1759 \pmod{7980}$	10	$k \equiv 524 \pmod{22344}$	1
$k \equiv 259 \pmod{2280}$	1	$k \equiv 14660 \pmod{15960}$	1	$k \equiv 3716 \pmod{22344}$	3
$k \equiv 1399 \pmod{2280}$	2	$k \equiv 8276 \pmod{15960}$	2	$k \equiv 6908 \pmod{22344}$	4
$k \equiv 105 \pmod{133}$	1	$k \equiv 1892 \pmod{15960}$	3	$k \equiv 126 \pmod{399}$	1
$k \equiv 29 \pmod{133}$	2	$k \equiv 11468 \pmod{15960}$	4	$k \equiv 183 \pmod{399}$	2
$k \equiv 86 \pmod{133}$	3	$k \equiv 4419 \pmod{5985}$	1	$k \equiv 240 \pmod{399}$	3
$k \equiv 143 \pmod{266}$	1	$k \equiv 2424 \pmod{5985}$	2	$k \equiv 297 \pmod{399}$	4
$k \equiv 257 \pmod{266}$	2	$k \equiv 429 \pmod{5985}$	3	$k \equiv 12 \pmod{399}$	5
$k \equiv 181 \pmod{266}$	3	$k \equiv 486 \pmod{1197}$	1	$k \equiv 468 \pmod{798}$	1

Table 9: Covering information for $d = -3$ (Part IV)

congruence	p
$k \equiv 69 \pmod{798}$	2
$k \equiv 734 \pmod{798}$	3
$k \equiv 1190 \pmod{1596}$	1
$k \equiv 50 \pmod{1596}$	2
$k \equiv 506 \pmod{1596}$	3
$k \equiv 962 \pmod{1596}$	4
$k \equiv 1988 \pmod{3192}$	1
$k \equiv 2444 \pmod{3192}$	2
$k \equiv 2900 \pmod{3192}$	3
$k \equiv 164 \pmod{6384}$	1
$k \equiv 3356 \pmod{6384}$	2
$k \equiv 1874 \pmod{4788}$	1
$k \equiv 3470 \pmod{4788}$	2
$k \equiv 278 \pmod{4788}$	3
$k \equiv 2348 \pmod{4788}$	4
$k \equiv 4268 \pmod{9576}$	1
$k \equiv 7460 \pmod{9576}$	2
$k \equiv 1076 \pmod{9576}$	3
$k \equiv 9530 \pmod{9576}$	4
$k \equiv 4742 \pmod{9576}$	5
$k \equiv 108 \pmod{513}$	1
$k \equiv 279 \pmod{513}$	2
$k \equiv 963 \pmod{1026}$	1
$k \equiv 165 \pmod{1026}$	2
$k \equiv 849 \pmod{1026}$	3
$k \equiv 507 \pmod{1026}$	4
$k \equiv 18 \pmod{108}$	1
$k \equiv 30 \pmod{108}$	2
$k \equiv 66 \pmod{108}$	3
$k \equiv 2046 \pmod{2052}$	1
$k \equiv 1476 \pmod{4104}$	1
$k \equiv 3756 \pmod{4104}$	2
$k \equiv 2388 \pmod{8208}$	1
$k \equiv 6492 \pmod{8208}$	2
$k \equiv 5124 \pmod{8208}$	3
$k \equiv 1020 \pmod{8208}$	4
$k \equiv 60 \pmod{135}$	1
$k \equiv 6 \pmod{135}$	2
$k \equiv 87 \pmod{135}$	3
$k \equiv 33 \pmod{135}$	4
$k \equiv 114 \pmod{135}$	5
$k \equiv 2445 \pmod{2565}$	1
$k \equiv 906 \pmod{2565}$	2
$k \equiv 1932 \pmod{2565}$	3
$k \equiv 393 \pmod{2565}$	4
$k \equiv 69 \pmod{270}$	1
$k \equiv 105 \pmod{270}$	2
$k \equiv 51 \pmod{270}$	3
$k \equiv 267 \pmod{270}$	4
$k \equiv 2103 \pmod{5130}$	1

congruence	p
$k \equiv 3129 \pmod{5130}$	2
$k \equiv 474 \pmod{540}$	1
$k \equiv 510 \pmod{540}$	2
$k \equiv 186 \pmod{540}$	3
$k \equiv 402 \pmod{540}$	4
$k \equiv 78 \pmod{540}$	5
$k \equiv 294 \pmod{540}$	6
$k \equiv 204 \pmod{540}$	7
$k \equiv 6720 \pmod{10260}$	2
$k \equiv 2616 \pmod{10260}$	1
$k \equiv 8772 \pmod{10260}$	3
$k \equiv 4668 \pmod{10260}$	4
$k \equiv 564 \pmod{10260}$	5
$k \equiv 2 \pmod{48}$	1
$k \equiv 38 \pmod{96}$	1
$k \equiv 86 \pmod{96}$	2
$k \equiv 74 \pmod{96}$	3
$k \equiv 26 \pmod{96}$	4
$k \equiv 14 \pmod{192}$	1
$k \equiv 158 \pmod{192}$	2
$k \equiv 110 \pmod{192}$	3
$k \equiv 350 \pmod{480}$	1
$k \equiv 446 \pmod{480}$	2
$k \equiv 62 \pmod{480}$	3
$k \equiv 638 \pmod{960}$	1
$k \equiv 254 \pmod{960}$	2
$k \equiv 1153 \pmod{1824}$	1
$k \equiv 241 \pmod{1824}$	2
$k \equiv 1381 \pmod{1824}$	3
$k \equiv 469 \pmod{1824}$	4
$k \equiv 1609 \pmod{1824}$	5
$k \equiv 2521 \pmod{3648}$	1
$k \equiv 697 \pmod{3648}$	2
$k \equiv 45 \pmod{80}$	1
$k \equiv 61 \pmod{80}$	2
$k \equiv 157 \pmod{240}$	1
$k \equiv 13 \pmod{240}$	2
$k \equiv 109 \pmod{240}$	3
$k \equiv 127 \pmod{228}$	7
$k \equiv 20 \pmod{120}$	1
$k \equiv 716 \pmod{2280}$	3
$k \equiv 1172 \pmod{2280}$	4
$k \equiv 3908 \pmod{4560}$	1
$k \equiv 1628 \pmod{4560}$	2
$k \equiv 2084 \pmod{4560}$	3
$k \equiv 4364 \pmod{4560}$	4
$k \equiv 166 \pmod{209}$	1
$k \equiv 90 \pmod{209}$	2
$k \equiv 147 \pmod{209}$	3
$k \equiv 71 \pmod{209}$	4

congruence	p
$k \equiv 204 \pmod{209}$	5
$k \equiv 337 \pmod{418}$	1
$k \equiv 261 \pmod{418}$	2
$k \equiv 185 \pmod{418}$	3
$k \equiv 109 \pmod{418}$	4
$k \equiv 546 \pmod{836}$	1
$k \equiv 470 \pmod{836}$	2
$k \equiv 394 \pmod{836}$	3
$k \equiv 1154 \pmod{3344}$	1
$k \equiv 1990 \pmod{3344}$	2
$k \equiv 2826 \pmod{3344}$	3
$k \equiv 318 \pmod{3344}$	4
$k \equiv 964 \pmod{1672}$	1
$k \equiv 52 \pmod{1672}$	2
$k \equiv 812 \pmod{1672}$	3
$k \equiv 1572 \pmod{1672}$	4
$k \equiv 243 \pmod{627}$	1
$k \equiv 585 \pmod{627}$	2
$k \equiv 15 \pmod{627}$	3
$k \equiv 357 \pmod{627}$	4
$k \equiv 72 \pmod{2508}$	1
$k \equiv 1953 \pmod{2508}$	2
$k \equiv 8850 \pmod{10032}$	1
$k \equiv 6342 \pmod{10032}$	2
$k \equiv 3834 \pmod{10032}$	3
$k \equiv 1326 \pmod{10032}$	4
$k \equiv 15747 \pmod{20064}$	1
$k \equiv 10731 \pmod{20064}$	2
$k \equiv 5715 \pmod{20064}$	3
$k \equiv 699 \pmod{20064}$	4
$k \equiv 3207 \pmod{5016}$	1
$k \equiv 452 \pmod{5016}$	2
$k \equiv 4556 \pmod{5016}$	3
$k \equiv 2732 \pmod{5016}$	4
$k \equiv 1820 \pmod{5016}$	5
$k \equiv 908 \pmod{5016}$	6
$k \equiv 661 \pmod{1254}$	1
$k \equiv 1003 \pmod{1254}$	2
$k \equiv 433 \pmod{1254}$	3
$k \equiv 775 \pmod{1254}$	4
$k \equiv 1117 \pmod{1254}$	5
$k \equiv 205 \pmod{1045}$	1
$k \equiv 1041 \pmod{1045}$	2
$k \equiv 832 \pmod{1045}$	3
$k \equiv 623 \pmod{2090}$	1
$k \equiv 1459 \pmod{2090}$	2
$k \equiv 965 \pmod{2090}$	3
$k \equiv 1801 \pmod{4180}$	1
$k \equiv 3891 \pmod{4180}$	2
$k \equiv 3758 \pmod{4180}$	3

Table 9: Covering information for $d = -3$ (Part V)

congruence	p
$k \equiv 414 \pmod{4180}$	4
$k \equiv 2010 \pmod{4180}$	5
$k \equiv 2846 \pmod{4180}$	6
$k \equiv 1668 \pmod{8360}$	1
$k \equiv 6684 \pmod{8360}$	2
$k \equiv 4100 \pmod{8360}$	3
$k \equiv 756 \pmod{16720}$	1
$k \equiv 9116 \pmod{16720}$	2
$k \equiv 2637 \pmod{3135}$	1
$k \equiv 1383 \pmod{3135}$	4
$k \equiv 129 \pmod{3135}$	5
$k \equiv 1725 \pmod{3135}$	2
$k \equiv 471 \pmod{3135}$	3
$k \equiv 2352 \pmod{3135}$	6
$k \equiv 1098 \pmod{3135}$	7
$k \equiv 2979 \pmod{3135}$	8
$k \equiv 1440 \pmod{3135}$	9
$k \equiv 547 \pmod{6270}$	1
$k \equiv 5563 \pmod{6270}$	2
$k \equiv 4309 \pmod{6270}$	3
$k \equiv 625 \pmod{660}$	1
$k \equiv 295 \pmod{660}$	2
$k \equiv 361 \pmod{660}$	5
$k \equiv 31 \pmod{660}$	3
$k \equiv 97 \pmod{660}$	4
$k \equiv 427 \pmod{660}$	6
$k \equiv 493 \pmod{660}$	7
$k \equiv 163 \pmod{660}$	8
$k \equiv 889 \pmod{12540}$	1
$k \equiv 7159 \pmod{12540}$	2
$k \equiv 2485 \pmod{12540}$	4
$k \equiv 8755 \pmod{12540}$	3
$k \equiv 20 \pmod{1320}$	1
$k \equiv 1076 \pmod{1320}$	2
$k \equiv 812 \pmod{1320}$	3
$k \equiv 548 \pmod{1320}$	4
$k \equiv 284 \pmod{1320}$	5
$k \equiv 932 \pmod{1320}$	6
$k \equiv 668 \pmod{1320}$	7
$k \equiv 404 \pmod{1320}$	8
$k \equiv 22340 \pmod{25080}$	1
$k \equiv 351 \pmod{495}$	1
$k \equiv 21 \pmod{495}$	2
$k \equiv 186 \pmod{495}$	3
$k \equiv 252 \pmod{495}$	4
$k \equiv 417 \pmod{495}$	5
$k \equiv 87 \pmod{495}$	6
$k \equiv 7083 \pmod{9405}$	1
$k \equiv 813 \pmod{9405}$	4
$k \equiv 3948 \pmod{9405}$	2

congruence	p
$k \equiv 8964 \pmod{9405}$	3
$k \equiv 2694 \pmod{9405}$	5
$k \equiv 5829 \pmod{9405}$	6
$k \equiv 901 \pmod{990}$	1
$k \equiv 571 \pmod{990}$	2
$k \equiv 241 \pmod{990}$	3
$k \equiv 307 \pmod{990}$	4
$k \equiv 18787 \pmod{18810}$	1
$k \equiv 12517 \pmod{18810}$	2
$k \equiv 17533 \pmod{18810}$	3
$k \equiv 373 \pmod{1980}$	1
$k \equiv 1363 \pmod{1980}$	3
$k \equiv 1033 \pmod{1980}$	6
$k \equiv 43 \pmod{1980}$	5
$k \equiv 109 \pmod{1980}$	4
$k \equiv 1099 \pmod{1980}$	2
$k \equiv 769 \pmod{1980}$	7
$k \equiv 1759 \pmod{1980}$	8
$k \equiv 35089 \pmod{37620}$	2
$k \equiv 16279 \pmod{37620}$	1
$k \equiv 956 \pmod{3960}$	1
$k \equiv 2276 \pmod{3960}$	2
$k \equiv 3596 \pmod{3960}$	3
$k \equiv 3332 \pmod{3960}$	4
$k \equiv 692 \pmod{3960}$	5
$k \equiv 2012 \pmod{3960}$	6
$k \equiv 37388 \pmod{75240}$	1
$k \equiv 62468 \pmod{75240}$	2
$k \equiv 12308 \pmod{75240}$	3
$k \equiv 67484 \pmod{75240}$	4
$k \equiv 17324 \pmod{75240}$	5
$k \equiv 42404 \pmod{75240}$	6
$k \equiv 130 \pmod{247}$	1
$k \equiv 92 \pmod{247}$	3
$k \equiv 54 \pmod{247}$	2
$k \equiv 16 \pmod{247}$	4
$k \equiv 225 \pmod{247}$	5
$k \equiv 187 \pmod{494}$	1
$k \equiv 149 \pmod{494}$	2
$k \equiv 111 \pmod{494}$	3
$k \equiv 73 \pmod{494}$	4
$k \equiv 434 \pmod{988}$	1
$k \equiv 890 \pmod{988}$	2
$k \equiv 358 \pmod{988}$	3
$k \equiv 2790 \pmod{2964}$	1
$k \equiv 814 \pmod{2964}$	2
$k \equiv 1802 \pmod{2964}$	3
$k \equiv 1916 \pmod{1976}$	1
$k \equiv 396 \pmod{1976}$	2
$k \equiv 852 \pmod{1976}$	3

congruence	p
$k \equiv 1308 \pmod{1976}$	4
$k \equiv 282 \pmod{741}$	1
$k \equiv 529 \pmod{741}$	2
$k \equiv 35 \pmod{741}$	3
$k \equiv 738 \pmod{741}$	4
$k \equiv 453 \pmod{741}$	5
$k \equiv 985 \pmod{1482}$	1
$k \equiv 1441 \pmod{1482}$	2
$k \equiv 4196 \pmod{5928}$	1
$k \equiv 4652 \pmod{5928}$	2
$k \equiv 415 \pmod{1235}$	1
$k \equiv 1156 \pmod{1235}$	2
$k \equiv 662 \pmod{1235}$	3
$k \equiv 168 \pmod{1235}$	4
$k \equiv 909 \pmod{1235}$	5
$k \equiv 29 \pmod{34}$	1
$k \equiv 128 \pmod{204}$	1
$k \equiv 17 \pmod{323}$	1
$k \equiv 188 \pmod{323}$	2
$k \equiv 36 \pmod{323}$	3
$k \equiv 530 \pmod{646}$	1
$k \equiv 207 \pmod{646}$	2
$k \equiv 55 \pmod{646}$	3
$k \equiv 549 \pmod{646}$	4
$k \equiv 397 \pmod{646}$	5
$k \equiv 245 \pmod{646}$	6
$k \equiv 93 \pmod{646}$	7
$k \equiv 378 \pmod{1292}$	1
$k \equiv 226 \pmod{1292}$	2
$k \equiv 74 \pmod{1292}$	3
$k \equiv 330 \pmod{340}$	1
$k \equiv 126 \pmod{340}$	2
$k \equiv 262 \pmod{340}$	3
$k \equiv 58 \pmod{340}$	4
$k \equiv 194 \pmod{340}$	5
$k \equiv 2354 \pmod{2584}$	1
$k \equiv 1062 \pmod{2584}$	2
$k \equiv 1746 \pmod{2584}$	5
$k \equiv 454 \pmod{2584}$	3
$k \equiv 2316 \pmod{2584}$	4
$k \equiv 2164 \pmod{2584}$	6
$k \equiv 2012 \pmod{2584}$	7
$k \equiv 1860 \pmod{2584}$	8
$k \equiv 4292 \pmod{5168}$	1
$k \equiv 1708 \pmod{5168}$	2
$k \equiv 3684 \pmod{10336}$	1
$k \equiv 1100 \pmod{10336}$	2
$k \equiv 8852 \pmod{10336}$	3
$k \equiv 6268 \pmod{10336}$	4
$k \equiv 264 \pmod{969}$	1

Table 9: Covering information for $d = -3$ (Part VI)

congruence	p
$k \equiv 435 \pmod{969}$	2
$k \equiv 606 \pmod{969}$	3
$k \equiv 948 \pmod{969}$	4
$k \equiv 2088 \pmod{2907}$	1
$k \equiv 1119 \pmod{2907}$	2
$k \equiv 150 \pmod{2907}$	3
$k \equiv 3228 \pmod{3876}$	1
$k \equiv 321 \pmod{3876}$	2
$k \equiv 1290 \pmod{3876}$	3
$k \equiv 2259 \pmod{3876}$	4
$k \equiv 1879 \pmod{1938}$	1
$k \equiv 1081 \pmod{1938}$	2
$k \equiv 283 \pmod{1938}$	3
$k \equiv 625 \pmod{1938}$	4
$k \equiv 1765 \pmod{1938}$	5
$k \equiv 4843 \pmod{5814}$	1
$k \equiv 967 \pmod{5814}$	2
$k \equiv 2905 \pmod{5814}$	3
$k \equiv 6572 \pmod{7752}$	1
$k \equiv 3836 \pmod{7752}$	2
$k \equiv 6116 \pmod{7752}$	4
$k \equiv 3380 \pmod{7752}$	3
$k \equiv 644 \pmod{7752}$	5
$k \equiv 815 \pmod{1615}$	1
$k \equiv 1461 \pmod{1615}$	2
$k \equiv 492 \pmod{3230}$	1
$k \equiv 2107 \pmod{3230}$	2
$k \equiv 1138 \pmod{3230}$	3
$k \equiv 2753 \pmod{6460}$	2
$k \equiv 5983 \pmod{6460}$	1
$k \equiv 1784 \pmod{6460}$	3
$k \equiv 169 \pmod{6460}$	4
$k \equiv 5014 \pmod{6460}$	5

congruence	p
$k \equiv 3399 \pmod{6460}$	6
$k \equiv 18 \pmod{361}$	1
$k \equiv 37 \pmod{361}$	3
$k \equiv 56 \pmod{361}$	2
$k \equiv 75 \pmod{361}$	4
$k \equiv 94 \pmod{361}$	5
$k \equiv 113 \pmod{361}$	6
$k \equiv 132 \pmod{361}$	7
$k \equiv 151 \pmod{722}$	1
$k \equiv 531 \pmod{722}$	2
$k \equiv 189 \pmod{722}$	3
$k \equiv 1234 \pmod{1444}$	1
$k \equiv 170 \pmod{1444}$	2
$k \equiv 550 \pmod{1444}$	3
$k \equiv 512 \pmod{1444}$	4
$k \equiv 892 \pmod{2888}$	1
$k \equiv 5604 \pmod{5776}$	1
$k \equiv 2716 \pmod{5776}$	2
$k \equiv 930 \pmod{1083}$	1
$k \equiv 588 \pmod{1083}$	2
$k \equiv 246 \pmod{1083}$	4
$k \equiv 987 \pmod{1083}$	3
$k \equiv 645 \pmod{1083}$	5
$k \equiv 303 \pmod{1083}$	6
$k \equiv 1044 \pmod{1083}$	7
$k \equiv 702 \pmod{3249}$	1
$k \equiv 1785 \pmod{3249}$	2
$k \equiv 2868 \pmod{3249}$	3
$k \equiv 1291 \pmod{2166}$	1
$k \equiv 949 \pmod{2166}$	2
$k \equiv 607 \pmod{2166}$	3
$k \equiv 265 \pmod{2166}$	4
$k \equiv 2089 \pmod{2166}$	5

congruence	p
$k \equiv 1747 \pmod{6498}$	1
$k \equiv 6079 \pmod{6498}$	2
$k \equiv 3913 \pmod{6498}$	3
$k \equiv 1405 \pmod{4332}$	1
$k \equiv 3571 \pmod{4332}$	2
$k \equiv 3229 \pmod{4332}$	3
$k \equiv 1063 \pmod{4332}$	4
$k \equiv 1652 \pmod{8664}$	1
$k \equiv 3476 \pmod{8664}$	2
$k \equiv 5300 \pmod{8664}$	3
$k \equiv 8948 \pmod{12996}$	1
$k \equiv 284 \pmod{12996}$	2
$k \equiv 4616 \pmod{12996}$	3
$k \equiv 2108 \pmod{12996}$	5
$k \equiv 6440 \pmod{12996}$	4
$k \equiv 10772 \pmod{12996}$	6
$k \equiv 8264 \pmod{12996}$	7
$k \equiv 12596 \pmod{12996}$	8
$k \equiv 3932 \pmod{25992}$	1
$k \equiv 14420 \pmod{25992}$	2
$k \equiv 5756 \pmod{25992}$	3
$k \equiv 23084 \pmod{25992}$	4
$k \equiv 15788 \pmod{25992}$	5
$k \equiv 24452 \pmod{25992}$	6
$k \equiv 7124 \pmod{51984}$	1
$k \equiv 33116 \pmod{51984}$	2
$k \equiv 360 \pmod{1805}$	1
$k \equiv 721 \pmod{1805}$	2
$k \equiv 1082 \pmod{1805}$	3
$k \equiv 1443 \pmod{1805}$	4
$k \equiv 1804 \pmod{1805}$	5

Table 10: Covering information for $d = -2$ (Part I)

congruence	p
$k \equiv 0 \pmod{2}$	1
$k \equiv 1 \pmod{16}$	1
$k \equiv 8 \pmod{15}$	1
$k \equiv 0 \pmod{23}$	1
$k \equiv 1 \pmod{345}$	1
$k \equiv 70 \pmod{345}$	2
$k \equiv 116 \pmod{345}$	3
$k \equiv 185 \pmod{345}$	4
$k \equiv 231 \pmod{345}$	5
$k \equiv 300 \pmod{345}$	6
$k \equiv 47 \pmod{690}$	1
$k \equiv 93 \pmod{690}$	2
$k \equiv 277 \pmod{690}$	3

congruence	p
$k \equiv 507 \pmod{690}$	4
$k \equiv 553 \pmod{690}$	5
$k \equiv 139 \pmod{1380}$	1
$k \equiv 369 \pmod{1380}$	2
$k \equiv 599 \pmod{1380}$	3
$k \equiv 829 \pmod{1380}$	4
$k \equiv 1059 \pmod{1380}$	5
$k \equiv 1289 \pmod{1380}$	6
$k \equiv 3 \pmod{46}$	1
$k \equiv 5 \pmod{46}$	2
$k \equiv 27 \pmod{46}$	3
$k \equiv 29 \pmod{46}$	4
$k \equiv 7 \pmod{92}$	1

congruence	p
$k \equiv 53 \pmod{92}$	2
$k \equiv 77 \pmod{92}$	3
$k \equiv 31 \pmod{184}$	1
$k \equiv 123 \pmod{184}$	2
$k \equiv 9 \pmod{69}$	1
$k \equiv 32 \pmod{69}$	2
$k \equiv 55 \pmod{69}$	3
$k \equiv 33 \pmod{138}$	1
$k \equiv 79 \pmod{138}$	2
$k \equiv 125 \pmod{138}$	3
$k \equiv 57 \pmod{276}$	1
$k \equiv 103 \pmod{276}$	2
$k \equiv 149 \pmod{276}$	3

Table 10: Covering information for $d = -2$ (Part II)

congruence	p
$k \equiv 195 \pmod{276}$	4
$k \equiv 241 \pmod{276}$	5
$k \equiv 11 \pmod{552}$	1
$k \equiv 287 \pmod{552}$	2
$k \equiv 12 \pmod{115}$	1
$k \equiv 35 \pmod{115}$	2
$k \equiv 58 \pmod{115}$	3
$k \equiv 81 \pmod{115}$	4
$k \equiv 104 \pmod{115}$	5
$k \equiv 105 \pmod{230}$	1
$k \equiv 151 \pmod{230}$	2
$k \equiv 13 \pmod{460}$	1
$k \equiv 59 \pmod{460}$	2
$k \equiv 197 \pmod{460}$	3
$k \equiv 243 \pmod{460}$	4
$k \equiv 289 \pmod{460}$	5
$k \equiv 427 \pmod{460}$	6
$k \equiv 14 \pmod{161}$	1
$k \equiv 37 \pmod{161}$	2
$k \equiv 60 \pmod{161}$	3
$k \equiv 106 \pmod{161}$	4
$k \equiv 129 \pmod{161}$	5
$k \equiv 313 \pmod{1288}$	1
$k \equiv 405 \pmod{1288}$	2
$k \equiv 635 \pmod{1288}$	3
$k \equiv 957 \pmod{1288}$	5
$k \equiv 1049 \pmod{1288}$	4
$k \equiv 1279 \pmod{1288}$	6
$k \equiv 83 \pmod{644}$	1
$k \equiv 15 \pmod{322}$	1
$k \equiv 61 \pmod{322}$	2
$k \equiv 107 \pmod{322}$	3
$k \equiv 199 \pmod{322}$	4
$k \equiv 245 \pmod{322}$	5
$k \equiv 291 \pmod{322}$	6
$k \equiv 153 \pmod{644}$	2
$k \equiv 475 \pmod{644}$	3
$k \equiv 85 \pmod{483}$	1
$k \equiv 154 \pmod{483}$	2
$k \equiv 246 \pmod{483}$	3
$k \equiv 315 \pmod{483}$	4
$k \equiv 407 \pmod{483}$	5
$k \equiv 476 \pmod{483}$	6
$k \equiv 177 \pmod{483}$	7
$k \equiv 499 \pmod{1932}$	1
$k \equiv 821 \pmod{1932}$	2
$k \equiv 1465 \pmod{1932}$	3
$k \equiv 1787 \pmod{1932}$	4
$k \equiv 1235 \pmod{1288}$	7
$k \equiv 39 \pmod{368}$	1
$k \equiv 85 \pmod{368}$	2
$k \equiv 269 \pmod{2576}$	1
$k \equiv 591 \pmod{2576}$	2
$k \equiv 2201 \pmod{2576}$	3
$k \equiv 39 \pmod{966}$	1
$k \equiv 361 \pmod{966}$	2
$k \equiv 683 \pmod{966}$	3
$k \equiv 131 \pmod{805}$	1
$k \equiv 292 \pmod{805}$	2
$k \equiv 453 \pmod{805}$	3
$k \equiv 775 \pmod{805}$	4
$k \equiv 1419 \pmod{1610}$	1
$k \equiv 223 \pmod{1610}$	2
$k \equiv 545 \pmod{1610}$	3
$k \equiv 867 \pmod{1610}$	4
$k \equiv 1189 \pmod{1610}$	5
$k \equiv 1511 \pmod{1610}$	6
$k \equiv 63 \pmod{207}$	1
$k \equiv 109 \pmod{207}$	2
$k \equiv 155 \pmod{207}$	3
$k \equiv 201 \pmod{414}$	1
$k \equiv 247 \pmod{414}$	2
$k \equiv 293 \pmod{414}$	3
$k \equiv 339 \pmod{414}$	4
$k \equiv 385 \pmod{414}$	5
$k \equiv 17 \pmod{828}$	1
$k \equiv 431 \pmod{828}$	2
$k \equiv 110 \pmod{253}$	1
$k \equiv 133 \pmod{253}$	2
$k \equiv 156 \pmod{253}$	3
$k \equiv 179 \pmod{253}$	4
$k \equiv 202 \pmod{253}$	5
$k \equiv 225 \pmod{506}$	1
$k \equiv 271 \pmod{506}$	2
$k \equiv 501 \pmod{506}$	3
$k \equiv 41 \pmod{1012}$	1
$k \equiv 317 \pmod{1012}$	2
$k \equiv 547 \pmod{1012}$	3
$k \equiv 823 \pmod{2024}$	1
$k \equiv 1835 \pmod{2024}$	2
$k \equiv 87 \pmod{759}$	1
$k \equiv 340 \pmod{759}$	2
$k \equiv 593 \pmod{759}$	3
$k \equiv 65 \pmod{299}$	1
$k \equiv 157 \pmod{299}$	2
$k \equiv 249 \pmod{299}$	3
$k \equiv 341 \pmod{598}$	1
$k \equiv 433 \pmod{598}$	2
$k \equiv 525 \pmod{598}$	3
$k \equiv 19 \pmod{1196}$	2
$k \equiv 111 \pmod{1196}$	1
$k \equiv 203 \pmod{1196}$	3
$k \equiv 617 \pmod{1196}$	4
$k \equiv 709 \pmod{1196}$	5
$k \equiv 801 \pmod{1196}$	6
$k \equiv 893 \pmod{1196}$	7
$k \equiv 295 \pmod{2392}$	1
$k \equiv 1491 \pmod{2392}$	2
$k \equiv 88 \pmod{897}$	2
$k \equiv 387 \pmod{897}$	3
$k \equiv 686 \pmod{897}$	1
$k \equiv 479 \pmod{1794}$	1
$k \equiv 571 \pmod{1794}$	2
$k \equiv 1077 \pmod{1794}$	3
$k \equiv 1169 \pmod{1794}$	4
$k \equiv 1675 \pmod{1794}$	5
$k \equiv 1767 \pmod{1794}$	6
$k \equiv 117 \pmod{552}$	3
$k \equiv 393 \pmod{552}$	4
$k \equiv 531 \pmod{2208}$	1
$k \equiv 807 \pmod{2208}$	2
$k \equiv 1359 \pmod{2208}$	3
$k \equiv 1635 \pmod{2208}$	4
$k \equiv 1911 \pmod{2208}$	6
$k \equiv 2187 \pmod{2208}$	7
$k \equiv 255 \pmod{4416}$	1
$k \equiv 1083 \pmod{4416}$	2
$k \equiv 2463 \pmod{4416}$	3
$k \equiv 3291 \pmod{4416}$	4
$k \equiv 25 \pmod{1104}$	1
$k \equiv 163 \pmod{1104}$	2
$k \equiv 301 \pmod{1104}$	3
$k \equiv 439 \pmod{1104}$	4
$k \equiv 577 \pmod{1104}$	5
$k \equiv 715 \pmod{1104}$	6
$k \equiv 853 \pmod{1104}$	7
$k \equiv 2095 \pmod{4416}$	6
$k \equiv 4303 \pmod{4416}$	5
$k \equiv 991 \pmod{2208}$	5
$k \equiv 2 \pmod{621}$	1
$k \equiv 140 \pmod{621}$	2
$k \equiv 278 \pmod{621}$	3
$k \equiv 416 \pmod{621}$	4
$k \equiv 71 \pmod{1242}$	1
$k \equiv 209 \pmod{1242}$	2
$k \equiv 347 \pmod{1242}$	3
$k \equiv 1175 \pmod{1242}$	4
$k \equiv 485 \pmod{2484}$	1
$k \equiv 1727 \pmod{2484}$	2
$k \equiv 20 \pmod{391}$	1

Table 10: Covering information for $d = -2$ (Part III)

congruence	p
$k \equiv 204 \pmod{391}$	2
$k \equiv 273 \pmod{391}$	3
$k \equiv 342 \pmod{391}$	4
$k \equiv 43 \pmod{782}$	1
$k \equiv 89 \pmod{782}$	2
$k \equiv 227 \pmod{782}$	3
$k \equiv 365 \pmod{782}$	4
$k \equiv 549 \pmod{782}$	5
$k \equiv 687 \pmod{782}$	6
$k \equiv 181 \pmod{1564}$	1
$k \equiv 503 \pmod{1564}$	2
$k \equiv 963 \pmod{1564}$	3
$k \equiv 1285 \pmod{1564}$	4
$k \equiv 2205 \pmod{6256}$	1
$k \equiv 3769 \pmod{6256}$	2
$k \equiv 5333 \pmod{6256}$	3
$k \equiv 1423 \pmod{1564}$	5
$k \equiv 319 \pmod{3128}$	1
$k \equiv 1101 \pmod{3128}$	2
$k \equiv 1883 \pmod{3128}$	3
$k \equiv 2665 \pmod{12512}$	1
$k \equiv 8921 \pmod{12512}$	2
$k \equiv 388 \pmod{1173}$	1
$k \equiv 779 \pmod{1173}$	2
$k \equiv 1170 \pmod{1173}$	3
$k \equiv 457 \pmod{2346}$	1
$k \equiv 1239 \pmod{2346}$	2
$k \equiv 848 \pmod{1173}$	4
$k \equiv 135 \pmod{2346}$	3
$k \equiv 1699 \pmod{2346}$	4
$k \equiv 917 \pmod{4692}$	1
$k \equiv 3263 \pmod{4692}$	2
$k \equiv 21 \pmod{437}$	1
$k \equiv 136 \pmod{437}$	2

congruence	p
$k \equiv 228 \pmod{437}$	3
$k \equiv 251 \pmod{437}$	4
$k \equiv 343 \pmod{437}$	5
$k \equiv 159 \pmod{874}$	1
$k \equiv 389 \pmod{874}$	2
$k \equiv 481 \pmod{874}$	3
$k \equiv 711 \pmod{874}$	4
$k \equiv 803 \pmod{874}$	5
$k \equiv 67 \pmod{1748}$	1
$k \equiv 297 \pmod{1748}$	2
$k \equiv 619 \pmod{1748}$	3
$k \equiv 941 \pmod{1748}$	4
$k \equiv 1493 \pmod{1748}$	5
$k \equiv 849 \pmod{3496}$	1
$k \equiv 1171 \pmod{3496}$	2
$k \equiv 1723 \pmod{3496}$	3
$k \equiv 2597 \pmod{3496}$	4
$k \equiv 2919 \pmod{3496}$	5
$k \equiv 3471 \pmod{3496}$	6
$k \equiv 90 \pmod{1311}$	1
$k \equiv 527 \pmod{1311}$	2
$k \equiv 964 \pmod{1311}$	3
$k \equiv 205 \pmod{2622}$	1
$k \equiv 757 \pmod{2622}$	2
$k \equiv 1079 \pmod{2622}$	3
$k \equiv 1631 \pmod{2622}$	4
$k \equiv 1953 \pmod{2622}$	5
$k \equiv 2505 \pmod{2622}$	6
$k \equiv 435 \pmod{2185}$	1
$k \equiv 872 \pmod{2185}$	2
$k \equiv 1309 \pmod{2185}$	3
$k \equiv 1746 \pmod{2185}$	4
$k \equiv 2183 \pmod{2185}$	5
$k \equiv 113 \pmod{4370}$	1

congruence	p
$k \equiv 987 \pmod{4370}$	2
$k \equiv 1861 \pmod{4370}$	3
$k \equiv 2735 \pmod{4370}$	4
$k \equiv 3609 \pmod{4370}$	5
$k \equiv 45 \pmod{460}$	7
$k \equiv 275 \pmod{460}$	8
$k \equiv 321 \pmod{1380}$	7
$k \equiv 781 \pmod{1380}$	9
$k \equiv 1241 \pmod{1380}$	8
$k \equiv 91 \pmod{920}$	1
$k \equiv 551 \pmod{920}$	2
$k \equiv 137 \pmod{1035}$	1
$k \equiv 252 \pmod{1035}$	2
$k \equiv 712 \pmod{1035}$	3
$k \equiv 367 \pmod{2070}$	1
$k \equiv 597 \pmod{2070}$	2
$k \equiv 827 \pmod{2070}$	3
$k \equiv 1057 \pmod{2070}$	4
$k \equiv 1517 \pmod{2070}$	5
$k \equiv 1977 \pmod{2070}$	6
$k \equiv 68 \pmod{575}$	1
$k \equiv 183 \pmod{575}$	2
$k \equiv 229 \pmod{575}$	3
$k \equiv 298 \pmod{575}$	4
$k \equiv 413 \pmod{575}$	5
$k \equiv 528 \pmod{575}$	6
$k \equiv 459 \pmod{1150}$	1
$k \equiv 689 \pmod{1150}$	2
$k \equiv 919 \pmod{2300}$	1
$k \equiv 1149 \pmod{2300}$	2
$k \equiv 2069 \pmod{2300}$	3
$k \equiv 2299 \pmod{2300}$	4

Table 11: Covering information for $d = 1$ (Part I)

congruence	p
$k \equiv 0 \pmod{7}$	1
$k \equiv 1 \pmod{7}$	2
$k \equiv 9 \pmod{21}$	1
$k \equiv 2 \pmod{21}$	2
$k \equiv 16 \pmod{21}$	3
$k \equiv 3 \pmod{14}$	1
$k \equiv 10 \pmod{28}$	1
$k \equiv 24 \pmod{28}$	2
$k \equiv 4 \pmod{35}$	1

congruence	p
$k \equiv 11 \pmod{35}$	2
$k \equiv 18 \pmod{35}$	3
$k \equiv 25 \pmod{70}$	1
$k \equiv 60 \pmod{70}$	2
$k \equiv 32 \pmod{105}$	1
$k \equiv 67 \pmod{105}$	2
$k \equiv 102 \pmod{105}$	3
$k \equiv 5 \pmod{42}$	1
$k \equiv 26 \pmod{42}$	2

congruence	p
$k \equiv 12 \pmod{42}$	3
$k \equiv 33 \pmod{84}$	1
$k \equiv 75 \pmod{84}$	2
$k \equiv 19 \pmod{63}$	1
$k \equiv 40 \pmod{63}$	2
$k \equiv 61 \pmod{63}$	3
$k \equiv 6 \pmod{28}$	3
$k \equiv 13 \pmod{56}$	1
$k \equiv 41 \pmod{56}$	2

Table 11: Covering information for $d = 1$ (Part II)

congruence	p
$k \equiv 20 \pmod{140}$	1
$k \equiv 48 \pmod{140}$	2
$k \equiv 76 \pmod{140}$	3
$k \equiv 104 \pmod{140}$	4

congruence	p
$k \equiv 132 \pmod{140}$	5
$k \equiv 27 \pmod{112}$	1
$k \equiv 83 \pmod{112}$	2
$k \equiv 55 \pmod{168}$	1

congruence	p
$k \equiv 111 \pmod{168}$	2
$k \equiv 167 \pmod{168}$	3

Table 12: Covering information for $d = 3$ (Part I)

congruence	p
$k \equiv 0 \pmod{3}$	1
$k \equiv 1 \pmod{6}$	2
$k \equiv 2 \pmod{6}$	1
$k \equiv 4 \pmod{12}$	1
$k \equiv 10 \pmod{24}$	1
$k \equiv 22 \pmod{72}$	1
$k \equiv 46 \pmod{72}$	2
$k \equiv 70 \pmod{72}$	3
$k \equiv 3 \pmod{22}$	1
$k \equiv 12 \pmod{13}$	1
$k \equiv 11 \pmod{28}$	1
$k \equiv 5 \pmod{60}$	1
$k \equiv 0 \pmod{37}$	1
$k \equiv 1 \pmod{37}$	2
$k \equiv 2 \pmod{37}$	3
$k \equiv 3 \pmod{74}$	1
$k \equiv 41 \pmod{74}$	2
$k \equiv 5 \pmod{74}$	3
$k \equiv 80 \pmod{111}$	1
$k \equiv 44 \pmod{111}$	2
$k \equiv 8 \pmod{111}$	3
$k \equiv 83 \pmod{222}$	1
$k \equiv 47 \pmod{222}$	2
$k \equiv 11 \pmod{222}$	3
$k \equiv 197 \pmod{222}$	4
$k \equiv 13 \pmod{148}$	1
$k \equiv 87 \pmod{148}$	2
$k \equiv 125 \pmod{148}$	3
$k \equiv 51 \pmod{148}$	4
$k \equiv 89 \pmod{148}$	5
$k \equiv 15 \pmod{148}$	6
$k \equiv 53 \pmod{444}$	1
$k \equiv 275 \pmod{444}$	2
$k \equiv 17 \pmod{444}$	3
$k \equiv 239 \pmod{444}$	4
$k \equiv 129 \pmod{296}$	1
$k \equiv 203 \pmod{296}$	2
$k \equiv 277 \pmod{296}$	3
$k \equiv 647 \pmod{888}$	2
$k \equiv 833 \pmod{888}$	1

congruence	p
$k \equiv 611 \pmod{888}$	3
$k \equiv 389 \pmod{888}$	4
$k \equiv 167 \pmod{592}$	1
$k \equiv 463 \pmod{592}$	2
$k \equiv 353 \pmod{1776}$	1
$k \equiv 1241 \pmod{1776}$	2
$k \equiv 1685 \pmod{1776}$	3
$k \equiv 797 \pmod{1776}$	4
$k \equiv 1019 \pmod{1332}$	1
$k \equiv 131 \pmod{1332}$	2
$k \equiv 575 \pmod{1332}$	3
$k \equiv 1133 \pmod{1332}$	4
$k \equiv 467 \pmod{1332}$	5
$k \equiv 317 \pmod{333}$	1
$k \equiv 95 \pmod{333}$	2
$k \equiv 206 \pmod{333}$	3
$k \equiv 281 \pmod{333}$	4
$k \equiv 59 \pmod{666}$	1
$k \equiv 503 \pmod{666}$	2
$k \equiv 245 \pmod{666}$	3
$k \equiv 23 \pmod{666}$	4
$k \equiv 659 \pmod{1332}$	6
$k \equiv 61 \pmod{185}$	1
$k \equiv 172 \pmod{185}$	2
$k \equiv 98 \pmod{185}$	3
$k \equiv 24 \pmod{185}$	4
$k \equiv 321 \pmod{370}$	1
$k \equiv 247 \pmod{370}$	2
$k \equiv 173 \pmod{555}$	1
$k \equiv 284 \pmod{555}$	2
$k \equiv 26 \pmod{555}$	3
$k \equiv 137 \pmod{555}$	4
$k \equiv 248 \pmod{555}$	5
$k \equiv 359 \pmod{555}$	6
$k \equiv 101 \pmod{1110}$	1
$k \equiv 767 \pmod{1110}$	2
$k \equiv 693 \pmod{740}$	1
$k \equiv 323 \pmod{740}$	2
$k \equiv 249 \pmod{740}$	3
$k \equiv 619 \pmod{740}$	4

congruence	p
$k \equiv 361 \pmod{740}$	5
$k \equiv 731 \pmod{740}$	7
$k \equiv 657 \pmod{740}$	6
$k \equiv 287 \pmod{740}$	8
$k \equiv 213 \pmod{740}$	9
$k \equiv 583 \pmod{740}$	10
$k \equiv 875 \pmod{2220}$	1
$k \equiv 395 \pmod{2220}$	2
$k \equiv 2135 \pmod{2220}$	3
$k \equiv 1655 \pmod{2220}$	4
$k \equiv 1175 \pmod{2220}$	5
$k \equiv 1435 \pmod{1480}$	1
$k \equiv 695 \pmod{1480}$	2
$k \equiv 955 \pmod{1480}$	3
$k \equiv 215 \pmod{1480}$	4
$k \equiv 475 \pmod{1480}$	5
$k \equiv 4175 \pmod{4440}$	1
$k \equiv 2729 \pmod{4440}$	2
$k \equiv 1619 \pmod{4440}$	3
$k \equiv 509 \pmod{4440}$	4
$k \equiv 3839 \pmod{4440}$	5
$k \equiv 1731 \pmod{2960}$	1
$k \equiv 2471 \pmod{2960}$	2
$k \equiv 251 \pmod{2960}$	3
$k \equiv 991 \pmod{2960}$	4
$k \equiv 547 \pmod{2960}$	5
$k \equiv 4247 \pmod{8880}$	1
$k \equiv 2027 \pmod{8880}$	2
$k \equiv 8687 \pmod{8880}$	3
$k \equiv 1361 \pmod{2664}$	1
$k \equiv 473 \pmod{2664}$	2
$k \equiv 2249 \pmod{2664}$	3
$k \equiv 29 \pmod{2664}$	4
$k \equiv 4469 \pmod{5328}$	1
$k \equiv 1805 \pmod{5328}$	2
$k \equiv 917 \pmod{5328}$	3
$k \equiv 3581 \pmod{5328}$	4
$k \equiv 6023 \pmod{6660}$	1
$k \equiv 3803 \pmod{6660}$	2
$k \equiv 1583 \pmod{6660}$	3

Table 12: Covering information for $d = 3$ (Part II)

congruence	p
$k \equiv 29 \pmod{1665}$	1
$k \equiv 1139 \pmod{1665}$	2
$k \equiv 584 \pmod{1665}$	3
$k \equiv 1991 \pmod{3330}$	1
$k \equiv 3101 \pmod{3330}$	2
$k \equiv 881 \pmod{3330}$	3
$k \equiv 2657 \pmod{3330}$	4
$k \equiv 437 \pmod{3330}$	5
$k \equiv 1547 \pmod{3330}$	6
$k \equiv 178 \pmod{925}$	3
$k \equiv 733 \pmod{925}$	2
$k \equiv 363 \pmod{925}$	1
$k \equiv 918 \pmod{925}$	5
$k \equiv 548 \pmod{925}$	4
$k \equiv 104 \pmod{925}$	6
$k \equiv 659 \pmod{925}$	7
$k \equiv 289 \pmod{925}$	8
$k \equiv 844 \pmod{925}$	9
$k \equiv 1399 \pmod{1850}$	1
$k \equiv 401 \pmod{1850}$	2
$k \equiv 31 \pmod{1850}$	3
$k \equiv 1511 \pmod{1850}$	4
$k \equiv 1141 \pmod{1850}$	5
$k \equiv 2621 \pmod{2775}$	1
$k \equiv 2177 \pmod{2775}$	2
$k \equiv 2732 \pmod{2775}$	3
$k \equiv 512 \pmod{2775}$	5
$k \equiv 1067 \pmod{2775}$	4

congruence	p
$k \equiv 1622 \pmod{2775}$	6
$k \equiv 1178 \pmod{2775}$	7
$k \equiv 1733 \pmod{5550}$	1
$k \equiv 5063 \pmod{5550}$	2
$k \equiv 2843 \pmod{5550}$	3
$k \equiv 623 \pmod{5550}$	4
$k \equiv 179 \pmod{5550}$	5
$k \equiv 3509 \pmod{5550}$	6
$k \equiv 1289 \pmod{3700}$	1
$k \equiv 3139 \pmod{3700}$	2
$k \equiv 10169 \pmod{11100}$	4
$k \equiv 4619 \pmod{11100}$	1
$k \equiv 7949 \pmod{11100}$	3
$k \equiv 2399 \pmod{11100}$	2
$k \equiv 217 \pmod{259}$	1
$k \equiv 106 \pmod{259}$	2
$k \equiv 254 \pmod{259}$	3
$k \equiv 143 \pmod{259}$	4
$k \equiv 180 \pmod{259}$	5
$k \equiv 69 \pmod{518}$	1
$k \equiv 329 \pmod{518}$	2
$k \equiv 218 \pmod{777}$	1
$k \equiv 107 \pmod{777}$	2
$k \equiv 773 \pmod{777}$	3
$k \equiv 551 \pmod{777}$	4
$k \equiv 440 \pmod{777}$	5
$k \equiv 959 \pmod{1554}$	1
$k \equiv 71 \pmod{1554}$	3

congruence	p
$k \equiv 737 \pmod{1554}$	2
$k \equiv 1403 \pmod{1554}$	4
$k \equiv 1181 \pmod{1554}$	5
$k \equiv 293 \pmod{1554}$	6
$k \equiv 809 \pmod{1036}$	1
$k \equiv 921 \pmod{1036}$	2
$k \equiv 2069 \pmod{3108}$	1
$k \equiv 220 \pmod{407}$	1
$k \equiv 331 \pmod{407}$	2
$k \equiv 35 \pmod{407}$	3
$k \equiv 257 \pmod{407}$	4
$k \equiv 368 \pmod{407}$	5
$k \equiv 72 \pmod{407}$	6
$k \equiv 183 \pmod{814}$	1
$k \equiv 701 \pmod{814}$	3
$k \equiv 405 \pmod{814}$	2
$k \equiv 109 \pmod{814}$	4
$k \equiv 517 \pmod{814}$	5
$k \equiv 221 \pmod{1221}$	1
$k \equiv 332 \pmod{1221}$	2
$k \equiv 554 \pmod{1221}$	3
$k \equiv 665 \pmod{1221}$	4
$k \equiv 776 \pmod{1221}$	5
$k \equiv 887 \pmod{1221}$	6
$k \equiv 2219 \pmod{2442}$	1
$k \equiv 1109 \pmod{2442}$	2
$k \equiv 2441 \pmod{2442}$	3

Table 13: Covering information for $d = 4$

congruence	p
$k \equiv 1 \pmod{3}$	1
$k \equiv 0 \pmod{6}$	2
$k \equiv 5 \pmod{6}$	1
$k \equiv 2 \pmod{9}$	1
$k \equiv 14 \pmod{18}$	1
$k \equiv 3 \pmod{18}$	2
$k \equiv 17 \pmod{27}$	1
$k \equiv 8 \pmod{27}$	2
$k \equiv 26 \pmod{54}$	1

congruence	p
$k \equiv 53 \pmod{54}$	2
$k \equiv 15 \pmod{81}$	1
$k \equiv 33 \pmod{81}$	2
$k \equiv 51 \pmod{81}$	3
$k \equiv 69 \pmod{81}$	4
$k \equiv 6 \pmod{81}$	5
$k \equiv 105 \pmod{162}$	1
$k \equiv 123 \pmod{162}$	2
$k \equiv 141 \pmod{162}$	3

congruence	p
$k \equiv 159 \pmod{162}$	4
$k \equiv 9 \pmod{36}$	1
$k \equiv 27 \pmod{144}$	1
$k \equiv 99 \pmod{144}$	2
$k \equiv 63 \pmod{216}$	1
$k \equiv 135 \pmod{216}$	2
$k \equiv 207 \pmod{432}$	1
$k \equiv 423 \pmod{432}$	2

Table 14: Covering information for $d = 6$

congruence	p
$k \equiv 0 \pmod{5}$	1
$k \equiv 1 \pmod{5}$	2
$k \equiv 2 \pmod{10}$	1
$k \equiv 3 \pmod{20}$	1
$k \equiv 13 \pmod{20}$	2
$k \equiv 7 \pmod{30}$	1
$k \equiv 17 \pmod{30}$	2

congruence	p
$k \equiv 27 \pmod{30}$	3
$k \equiv 8 \pmod{40}$	2
$k \equiv 28 \pmod{40}$	1
$k \equiv 18 \pmod{60}$	1
$k \equiv 58 \pmod{60}$	2
$k \equiv 38 \pmod{60}$	3
$k \equiv 9 \pmod{15}$	1

congruence	p
$k \equiv 4 \pmod{15}$	2
$k \equiv 29 \pmod{45}$	1
$k \equiv 14 \pmod{45}$	2
$k \equiv 89 \pmod{90}$	1
$k \equiv 44 \pmod{90}$	2

Table 15: Covering information for $d = 7$

congruence	p
$k \equiv 16 \pmod{66}$	2
$k \equiv 136 \pmod{264}$	6
$k \equiv 202 \pmod{264}$	7
$k \equiv 70 \pmod{528}$	1
$k \equiv 268 \pmod{528}$	2
$k \equiv 334 \pmod{528}$	3
$k \equiv 532 \pmod{1584}$	1
$k \equiv 4 \pmod{4752}$	1
$k \equiv 1588 \pmod{4752}$	2
$k \equiv 3 \pmod{16}$	2
$k \equiv 11 \pmod{32}$	1
$k \equiv 27 \pmod{32}$	2
$k \equiv 0 \pmod{11}$	1
$k \equiv 1 \pmod{11}$	2
$k \equiv 10 \pmod{16}$	1
$k \equiv 2 \pmod{22}$	1
$k \equiv 13 \pmod{22}$	2
$k \equiv 3 \pmod{22}$	3
$k \equiv 14 \pmod{44}$	1
$k \equiv 36 \pmod{44}$	2
$k \equiv 15 \pmod{33}$	1
$k \equiv 26 \pmod{33}$	2
$k \equiv 37 \pmod{66}$	1
$k \equiv 38 \pmod{132}$	1
$k \equiv 126 \pmod{132}$	2
$k \equiv 60 \pmod{132}$	3
$k \equiv 5 \pmod{88}$	1
$k \equiv 49 \pmod{88}$	2
$k \equiv 104 \pmod{264}$	1
$k \equiv 236 \pmod{264}$	2
$k \equiv 71 \pmod{264}$	3
$k \equiv 159 \pmod{264}$	4
$k \equiv 247 \pmod{264}$	5
$k \equiv 6 \pmod{55}$	1
$k \equiv 17 \pmod{55}$	2
$k \equiv 28 \pmod{55}$	3
$k \equiv 39 \pmod{55}$	4
$k \equiv 50 \pmod{110}$	1
$k \equiv 105 \pmod{110}$	2
$k \equiv 7 \pmod{110}$	3
$k \equiv 62 \pmod{110}$	4
$k \equiv 18 \pmod{220}$	1
$k \equiv 73 \pmod{220}$	2
$k \equiv 128 \pmod{220}$	3
$k \equiv 183 \pmod{220}$	4
$k \equiv 40 \pmod{275}$	1

congruence	p
$k \equiv 95 \pmod{275}$	2
$k \equiv 150 \pmod{275}$	3
$k \equiv 29 \pmod{165}$	1
$k \equiv 84 \pmod{165}$	2
$k \equiv 139 \pmod{165}$	3
$k \equiv 51 \pmod{220}$	5
$k \equiv 161 \pmod{220}$	6
$k \equiv 106 \pmod{330}$	1
$k \equiv 216 \pmod{330}$	2
$k \equiv 326 \pmod{330}$	3
$k \equiv 8 \pmod{77}$	1
$k \equiv 19 \pmod{77}$	2
$k \equiv 30 \pmod{77}$	3
$k \equiv 41 \pmod{77}$	4
$k \equiv 52 \pmod{154}$	1
$k \equiv 129 \pmod{154}$	2
$k \equiv 63 \pmod{154}$	3
$k \equiv 140 \pmod{154}$	4
$k \equiv 74 \pmod{154}$	5
$k \equiv 151 \pmod{154}$	6
$k \equiv 9 \pmod{99}$	1
$k \equiv 20 \pmod{99}$	2
$k \equiv 31 \pmod{99}$	3
$k \equiv 42 \pmod{99}$	4
$k \equiv 53 \pmod{198}$	1
$k \equiv 152 \pmod{198}$	2
$k \equiv 64 \pmod{396}$	1
$k \equiv 163 \pmod{396}$	2
$k \equiv 262 \pmod{396}$	3
$k \equiv 361 \pmod{792}$	1
$k \equiv 757 \pmod{792}$	2
$k \equiv 75 \pmod{297}$	1
$k \equiv 174 \pmod{297}$	2
$k \equiv 273 \pmod{297}$	3
$k \equiv 86 \pmod{297}$	4
$k \equiv 185 \pmod{297}$	5
$k \equiv 284 \pmod{297}$	6
$k \equiv 97 \pmod{594}$	1
$k \equiv 196 \pmod{594}$	2
$k \equiv 295 \pmod{594}$	3
$k \equiv 394 \pmod{594}$	4
$k \equiv 493 \pmod{594}$	5
$k \equiv 592 \pmod{1188}$	1
$k \equiv 1186 \pmod{1188}$	2
$k \equiv 10 \pmod{121}$	1
$k \equiv 21 \pmod{121}$	2

congruence	p
$k \equiv 32 \pmod{121}$	3
$k \equiv 43 \pmod{121}$	4
$k \equiv 54 \pmod{242}$	1
$k \equiv 175 \pmod{242}$	2
$k \equiv 65 \pmod{242}$	3
$k \equiv 186 \pmod{242}$	4
$k \equiv 76 \pmod{242}$	5
$k \equiv 197 \pmod{484}$	1
$k \equiv 439 \pmod{484}$	2
$k \equiv 87 \pmod{484}$	3
$k \equiv 208 \pmod{484}$	4
$k \equiv 98 \pmod{363}$	1
$k \equiv 219 \pmod{363}$	2
$k \equiv 109 \pmod{726}$	1
$k \equiv 230 \pmod{726}$	2
$k \equiv 351 \pmod{726}$	3
$k \equiv 593 \pmod{1452}$	1
$k \equiv 1319 \pmod{4356}$	1
$k \equiv 2771 \pmod{4356}$	2
$k \equiv 2166 \pmod{5808}$	1
$k \equiv 5070 \pmod{5808}$	2
$k \equiv 2892 \pmod{5808}$	3
$k \equiv 5796 \pmod{5808}$	4
$k \equiv 120 \pmod{605}$	1
$k \equiv 362 \pmod{1210}$	1
$k \equiv 967 \pmod{1210}$	2
$k \equiv 1088 \pmod{2420}$	5
$k \equiv 2298 \pmod{2420}$	6
$k \equiv 604 \pmod{2420}$	1
$k \equiv 1209 \pmod{2420}$	2
$k \equiv 1814 \pmod{2420}$	3
$k \equiv 2419 \pmod{2420}$	4
$k \equiv 1060 \pmod{1584}$	2
$k \equiv 3172 \pmod{4752}$	3
$k \equiv 205 \pmod{275}$	4
$k \equiv 260 \pmod{275}$	5
$k \equiv 329 \pmod{484}$	5
$k \equiv 450 \pmod{484}$	6
$k \equiv 340 \pmod{363}$	3
$k \equiv 472 \pmod{726}$	4
$k \equiv 4223 \pmod{4356}$	3
$k \equiv 714 \pmod{2904}$	1
$k \equiv 1440 \pmod{2904}$	2
$k \equiv 241 \pmod{605}$	2
$k \equiv 483 \pmod{1210}$	3

Table 16: Covering information for $d = 9$

congruence	p
$k \equiv 0 \pmod{2}$	1
$k \equiv 3 \pmod{4}$	1

congruence	p
$k \equiv 1 \pmod{8}$	1

congruence	p
$k \equiv 5 \pmod{8}$	2