Abstracts for PANTS XXXV

Periodic points of an algebraic function related to a continued fraction of Ramanujan

Sushmanth Jacob Akkarapakam, Indiana University at Indianapolis

Abstract: A continued fraction $v(\tau)$ of Ramanujan is evaluated at certain arguments in the field $K = \mathbb{Q}(\sqrt{-d})$, with $-d \equiv 1 \pmod 8$, in which the ideal $(2) = \wp_2\wp_2'$ is a product of two prime ideals. These values of $v(\tau)$ are shown to generate the inertia field of $\wp_2$ or $\wp_2'$ in an extended ring class field over the field K. The same values are also shown to be periodic points of a fixed algebraic function $\hat{F}(x)$, independent of d. These are analogues of similar results for the Rogers-Ramanujan continued fraction.

Frobenius eigenvalues of abelian varieties over finite fields

Santiago Arango-Piñeros, Emory University

Abstract: Fix an abelian variety A over a finite field $\mathbf{F}_q$. For each degree r extension of the base field, let x_r denote the normalized trace of Frobenius of the base extension $A \times_{\mathbf{F}_q} \mathbf{F}_{q^r}$. In joint work-in-progress with Deewang Bhamidipati and Soumya Sankar, we study the distribution of the sequence (x_r) and give a (partial) classification in dimension $g \leq 3$.

Zeros of fractional derivatives of polynomials

Torre Caparatta, UNC Greensboro

Abstract: We study behavior of fractional derivatives $p^{(\alpha)}(x)$ of polynomials of degree n. First we find exact location of their zeros, and then we investigate the intriguing movement of these zeros and establish their convergence properties as alpha approaches n.

Computing isogeny classes of principally polarized abelian surfaces over the rationals

Edgar Costa, MIT

Abstract: Given a principally polarized abelian surface (PPAS) over the rationals, and would like to compute all the other PPASs in its isogeny class. We describe a practical algorithm to do this for PPAS with trivial endomorphism ring. This is joint work in progress with Raymond van Bommel, Shiva Chidambaram, and Jean Kieffer.

Representing positive integers as a sum of a square-free number and a small prime

Jack Dalton, University of South Carolina

Abstract: In 1931, Esterman showed that every sufficiently large integer can be written as the sum of a square-free number and a prime. In 2017, Dudek showed that this statement is true of all integers greater than 2. If a size restriction on the prime with respect to the original integer is introduced, we get finitely many exceptions. We prove that every positive integer which is not equal to $1, 2, 3, 6, 11, 30, 155$, or

247 can be represented as a sum of a prime not exceeding $\sqrt{n}$ and a square-free number. This is joint work with Ognian Trifonov.

Construction of polynomials with prescribed divisibility conditions on the critical orbit

Mohamed Wafik ElSheikh, University of South Carolina

Abstract: Let $f_{d,c}(x) = x^d + c \in \mathbb{Q}[x]$, $d \geq 2$. We write $f_{d,c}^n$ for $\underbrace{f_{d,c} \circ f_{d,c} \circ \cdots \circ f_{d,c}}_{\text{n times}}$. The critical orbit of $f_{d,c}(x)$ is the set $\mathcal{O}_{f_{d,c}}(0) := \{f_{d,c}^n(0) : n \geq 0\}$.

For a sequence $\{a_n : n \geq 0\}$, a primitive prime divisor for a_k is a prime dividing a_k but not a_n for any $1 \leq n < k$. A result of H. Krieger asserts that if the critical orbit $\mathcal{O}_{f_{d,c}}(0)$ is infinite, then each element in $\mathcal{O}_{f_{d,c}}(0)$ has at least one primitive prime divisor except possibly for 23 elements. In addition, under certain conditions, R. Jones proved that the density of primitive prime divisors appearing in any orbit of $f_{d,c}(x)$ is always 0.

In this talk, I'll discuss joint work with Mohammad Sadek, in which we display an upper bound on the count of primitive prime divisors of a fixed iteration $f_{d,c}^n(0)$. Further, we show that there is no uniform upper bound on the count of primitive prime divisors of $f_{d,c}^n(0)$ that does not depend on c. In particular, given $N > 0$, there is $c \in \mathbb{Q}$ such that $f_{d,c}^n(0)$ has at least N primitive prime divisors. This, along with some previous results, allows for the construction of polynomials of the form $f_{d,c}(x)$ whose n-th iterates possess maximal Galois Groups.

Bounds on torsion subgroups from geometric isogeny classes of elliptic curves

Tyler Genao, University of Georgia

Abstract: A celebrated result of Merel (1996) proved that for each integer $d \in \mathbb{Z}^+$ there exists a uniform bound on all torsion subgroups of elliptic curves over number fields of degree d; *a priori,* such elliptic curves have j-invariants of bounded degree over $\mathbb{Q}$. On the other hand, any $\overline{\mathbb{Q}}$-isogeny class of elliptic curves will contain j-invariants of arbitrarily large degree. It may be surprising, then, that torsion subgroups in these isogeny classes can still be "well-behaved" while allowing their number field degrees to vary. In this talk, we will discuss our new results on polynomial bounds for geometric isogeny classes of elliptic curves, as well as special asymptotic bounds (specifically, "typical boundedness") over the collection of geometric isogeny classes with at least one rational j-invariant.

Constructing generalized Sierpiński numbers

Robert Groth, University of South Carolina

Abstract: A Sierpiński number is a positive odd integer k such that $k \cdot 2^n + 1$ is composite for all $n \in \mathbb{Z}^+$. Fix an integer A with $2 \leq A$. We show that there exists a positive odd integer k such that $k \cdot a^n + 1$ is composite for all integers $a \in [2, A]$ and all $n \in \mathbb{Z}^+$. This is joint work with Michael Filaseta and Thomas Luckner.

Representation dimensions of algebraic tori

Bailey Heath, University of South Carolina

Abstract: Algebraic tori over a field k are special examples of affine group schemes over k, such as

the multiplicative group of the field or the unit circle. Any algebraic torus can be embedded into the group of $n \times n$ invertible matrices with entries in k for some n, and the smallest such n is called the representation dimension of that torus. In this work, I am interested in finding the smallest possible upper bound on the representation dimension of all algebraic tori of a given dimension d. After providing some background, I will discuss how we can rephrase this question in terms of finite groups of invertible integral matrices. Then, I will share some progress that I have made on this question, including exact answers for certain values of d.

Sums of four Pell numbers as powers of 3

Jacob Juillerat, University of North Carolina at Pembroke

Abstract: Let $(P_n)_{n\geq 0}$ be the Pell sequence defined by $P_0 = 1$, $P_1 = 1$, and $P_n = 2P_{n-1} + P_{n-2}$ for $n \geq 2$. Using lower bounds for the absolute value of linear forms in logarithms, a version of the Baker-Davenport reduction method, and properties of continued fractions of irrational numbers, we find all solutions to the Diophantine equation $P_{n_1} + P_{n_2} + P_{n_3} + P_{n_4} = 3^a$.

Categorifying zeta and L*-functions*

Andrew Kobin, Emory University

Abstract: Zeta and L-functions are ubiquitous in modern number theory. While some work in the past has brought homotopical methods into the theory of zeta functions, there is in fact a lesser-known zeta function that is native to homotopy theory. Namely, every suitably finite decomposition space (aka 2-Segal space) admits an abstract zeta function as an element of its incidence algebra. In this talk, I will show how the Dedekind zeta function of a number field can be realized in this homotopical framework. I will also discuss work in progress towards a categorification of L-functions (and beyond).

The distribution of short orbits of singular moduli

Riad Masri, Texas A&M University

Abstract: The values of the modular j-function at CM points are algebraic numbers called *singular moduli*. There is a natural action of the class group $G(d)$ of discriminant $d < 0$ on the set of singular moduli. For each d, fix a singular modulus $j(d, 0)$ and a subgroup $H(d) < G(d)$. Let $\mathrm{Av}(d, 0)$ denote the average of the numbers in the orbit $H(d) \cdot j(d, 0)$. We will show that under a mild growth condition on $|H(d)|$, one can compute the limiting distribution of $\mathrm{Av}(d, 0)$ as $|d| \to \infty$. This growth condition is expressed as a function of the best progress towards the Lindelöf hypothesis for modular L-functions.

An effective open image theorem for products of abelian varieties

Jacob Mayle, Wake Forest University

Abstract: In a celebrated 1972 article, Serre proved the open image theorem for elliptic curves asserting that the mod ℓ Galois representation of a non-CM elliptic curve is surjective provided that ℓ is sufficiently large. In the same article, he also considered the open image theorem for a product of non-CM elliptic curves. There are several effective bounds on the largest nonsurjective prime associated with an elliptic curve and a product of elliptic curves. We will discuss a conditional effective bound on the largest nonsurjective prime for a product of principally polarized abelian varieties, each with an open adelic Galois image. The bound is given in terms of conductors and the largest nonsurjective

prime of each abelian variety. This is an ongoing project in joint with Tian Wang.

Permutations and the divisor graph of $[1, n]$

Nathan McNew, Towson University

Abstract: Recently Pomerance asked how many permutations π of n satisfy a certain divisibility property, namely that either i divides $\pi(i)$ or $\pi(i)$ divides i for every index i. In a subsequent paper, he showed that the count is bounded between 1.93^n and 13.6^n. We give an improvement on this result, showing that the count is $(c + o(1))^n$ for a constant $2.06 < c < 2.70$. The proof uses bounds on the distribution of smooth numbers as well as a graph theoretic result on cycle-covers applied to the divisor graph (the graph on vertices $1, 2, \ldots, n$ and an edge between two integers if one divides the other). We also discuss related enumeration problems, such as counting primitive sets of the integers (subsets of the integers in which no element divides another) which can be approached using similar methods.

Rankin-Cohen brackets of vector valued Eisenstein series

Tianyu Ni, Clemson University

Abstract: We prove that the Rankin-Cohen brackets of two vector valued Eisentein series span the whole space of cuspforms as a Hecke module. This is an extension of Westerholt-Raum's work (2017) that products of two vector valued Eisenstein series span the whole space of cuspforms as a Hecke module. The idea is to use basic representation theory and Atkin-Lehner-Li theory to reduce the problem to the non-vanishing of special values of L-functions. This is joint work with Hui Xue.

A volcanic approach to CM points on Shimura curves

Frederick Saia, University of Georgia

Abstract: A CM component of the ℓ-isogeny graph of elliptic curves over a finite field has a particular structure, that of an ℓ-volcano, away from certain CM orders. The structure of these "isogeny volcanoes" has seen much use in the study of CM elliptic curves over finite fields, originating with the 1996 PhD thesis of Kohel. Recent work of Clark and Clark–Saia leverages infinite depth versions of these graphs to study moduli of isogenies of CM elliptic curves over $\overline{\mathbb{Q}}$. We will discuss an analogue of this work for abelian surfaces with quaternionic multiplication, sometimes referred to as "fake elliptic curves" A main result of our study is an algorithm for describing the Δ-CM locus on the Shimura curve $X_0^D(N)_{/\mathbb{Q}}$, for $\gcd(D, N) = 1$ and Δ any imaginary quadratic discriminant. Specifically, our work allows for an enumeration of all points in this locus with a given residue field and ramification index, generalizing a result from Jordan's 1981 thesis in the $N = 1$ case. As an application, we give an explicit list of pairs (N, D) for which the Shimura curve $X_0^D(N)_{/\mathbb{Q}}$ may fail to have a sporadic point.

Computational aspects of modular forms

Kalani Thalagoda, University of North Carolina Greensboro

Abstract: Modular forms over imaginary quadratic fields (Bianchi modular forms) are a generalization of classical modular forms. The modularity for Bianchi Modular forms is still conjectural. Thus, explicit computational evidence is helpful in understanding the connection between Bianchi modular forms and elliptic curves. In this talk, I present computational techniques we developed to compute Bianchi modular forms over imaginary quadratic fields with class number 4. I will also present some

interesting examples observed in the implementation of these techniques for $Q(\sqrt{-17})$. In particular, I will present the first known example of a modular elliptic curve over an imaginary quadratic field of class number 4. The work I present in this talk is joint work with Dan Yasaki.

Bounds on successive minima of orders in number fields and related counting problems

Sameera Vemulapalli, Princeton University

Abstract: Successive minima are real numbers associated to an order in a number field that encode information about its shape. Lenstra conjectured precise inequalities on successive minima of orders in number fields; we prove Lenstra's conjecture in many cases and prove that it is false in general. When $n < 6$, I'll also discuss a related counting question: how many orders in degree n number fields are there with bounded discriminant and prescribed successive minima?

Improved bounds on entanglement of torsion point fields

Giacomo Viazzo, Wake Forest University

Abstract: Serre's Open Image Theorem states that the image of the adelic Galois representation of a non-CM elliptic curve E over a number field k is open in $\mathrm{GL}_2(\hat{Z})$, and hence has finite index. Thus, the adelic image is completely determined by the image of the mod N Galois representation for a positive integer N, the smallest of which is called the level of the adelic image. Notably, while the theorem holds for a fixed non-CM elliptic curve, there is no uniform bound on the level when considering a collection of elliptic curves, as it can be arbitrarily large. Nevertheless, it has been shown that, for a fixed positive integer m, there is a uniform and explicit bound on the level of the m-adic Galois representation that applies for all non-CM elliptic curves over $\mathbb{Q}$. In this talk, we will show how introducing good reduction assumptions allows us to improve this uniform bound on the m-adic representation and, therefore, obtain more precise information on the entanglement of the torsion point fields.

Geometry of curves with abundant low degree points

Isabel Vogt, Brown University

Abstract: An important invariant of a curve defined over a number field is the minimal degree for which it has infinitely many closed points of that degree. Faltings' famous theorem characterizes when this invariant takes the value 1. In this talk I will discuss joint work with Borys Kadets, extending classification results of Harris–Silverman and Abramovich–Harris, in which we characterize when this invariant takes other small values.

Prime gaps and Siegel zeroes after Zhang

Tom Wright, Wofford College

Abstract: In 1983, Roger Heath-Brown proved that the existence of Siegel zeroes would imply the twin prime conjecture. In this talk, we show how one could generalize this result (using the methods of Maynard and Tao) to find that Siegel zeroes would imply narrow k-tuples of primes for any k. Along the way, we also improve bounds on some Siegel zero results to make them more resistant to the as-of-yet unverified breakthrough by Yitang Zhang.

A dynamical analogue of a question of Fermat

Tugba Yesin, Sabancı University

Abstract: The existence of three consecutive squares in the arithmetic progression is a phenomenon that can be seen in $\mathbb{Q}$. The elements 1, 5^2, and 7^2 provide such an example. Fermat claimed that there does not exist an arithmetic progression of four squares in $\mathbb{Q}$. Euler, among others, proved Fermat's claim. In this talk, I'll describe joint work with Mohammad Sadek, giving a dynamical analogue of Fermat's Squares Theorem. Namely, given a degree two polynomial $f(x) = x^2 + Ax + B \in K[x]$ and a point $x_0 \in K$, how many consecutive squares can be there in the orbit $\{x_0, f(x_0), f^2(x_0), \ldots, f^n(x_0), \ldots\}$ of x_0? We display three different constructions of 1-parameter quadratic polynomials with orbits containing three consecutive squares. In addition, we show that there exists at least one polynomial of the form $x^2 + c$ with a rational point whose orbit under this map contains four consecutive squares.