

MATH 374-001 Test 4

April 23, 2014

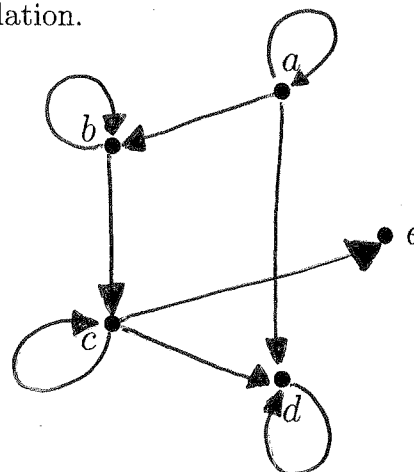
Name: Key

Directions: You have 50 minutes to complete the following exam. Show all applicable work. Answers without proper evidence of understanding will not receive credit.

1. Answer each question for the following binary relation on the set $S = \{a, b, c, d, e\}$.

$$\rho = \{(a, a), (a, b), (b, b), (a, d), (d, d), (c, d), (b, c), (c, c), (c, e)\}.$$

(2pt) (a) Draw the digraph that corresponds to the binary relation.



(1pt) (b) Is ρ antisymmetric?

Yes. (No arrows in both directions)

(1pt) (c) Is ρ symmetric?

No. (Doesn't have all arrows in both directions)

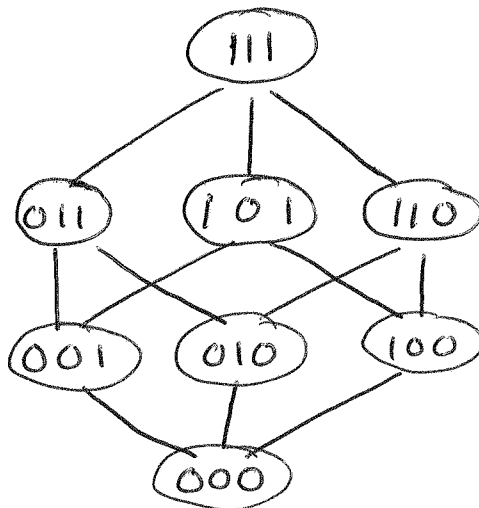
(1pt) (d) Is ρ reflexive?

No. ((e, e) is not in the relation)

(2pt) (e) Notice that ρ is not transitive. What needs to be added to make it transitive?

$$\{(a, c), (b, d), (b, e), (a, e)\}$$

(5pt) 2. Given the collection of all 0-1 strings of length 3, define a binary relation \preceq on them where $a \preceq b$ exactly when $a_i \leq b_i$ for every position. For example $010 \preceq 111$ (since $0 \leq 1, 1 \leq 1,$ and $0 \leq 1$), but $001 \not\preceq 010$ (because in the third position, $1 \not\leq 0$). This relation forms a poset. (You do not need to prove this.) Draw the Hasse diagram for this poset.



3. Define a binary relation ρ on the real numbers, \mathbb{R} , so that $x \rho y$ exactly when $x - y$ is an integer.

(5pt) (a) Show that ρ is an equivalence relation.

Reflexive: $x - x = 0$ is an integer, so $x \rho x$ always.

Symmetric If $x - y$ is an integer, so is $-(x - y) = y - x$.
So if $x \rho y$, then $y \rho x$.

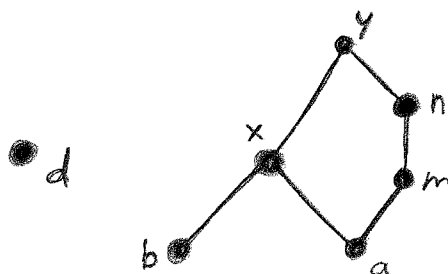
Transitive If $x \rho y$, and $y \rho z$, then $x - y$ and $y - z$ are both integers.
If you add these, you get another integer,
 $(x - y) + (y - z) = x - z$. Hence $x \rho z$.

(2pt) (b) Describe the equivalence class that contains the number 1.2. (That is, write it as a set.)

These are all y so that $1.2 - y$ is an integer. It's

$$\{ \dots, -3.8, -2.8, -1.8, -0.8, 0.2, 1.2, 2.2, 3.2, \dots \}$$

4. Below is a Hasse diagram for a poset.



(4pt) (a) Give a topological sort for the poset.

d, b, a, x, m, n, y

(1pt) (b) When is the latest that (d) can occur in a topological sort?

Last! It doesn't have any successors!

(i.e., b, a, x, m, n, y, d is alright).

(1pt) (c) When is the latest that (b) can occur in a topological sort?

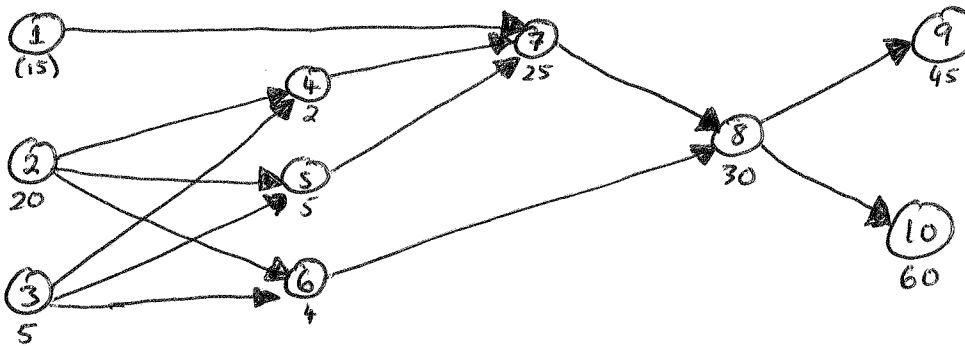
It must come before x, a, y . So, 3rd to last.

(i.e., d, a, m, n, b, x, y is alright).

5. The chart below lists steps in preparing a cake for a birthday party.

Task	Prerequisites	Minutes to perform
1. Preheat oven	none	15
2. Buy ingredients	none	20
3. Get out dishes	none	5
4. Grease cake pan	2,3	2
5. Mix batter	2,3	5
6. Mix icing	2,3	4
7. Pour/bake cake	1,4,5	25
8. Ice/Decorate	6,7	30
9. Sing and eat	8	45
10. Clean up the kitchen	8	60

(3pt) (a) Draw the PERT diagram.



(4pt) (b) Find the critical path, and the minimum amount of time required to finish all of the tasks.

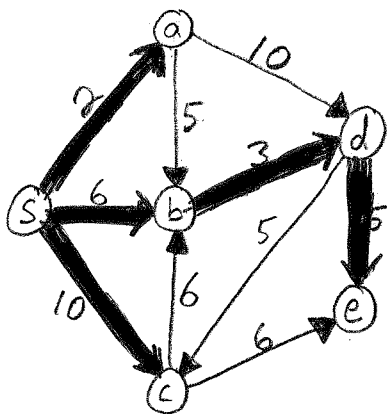
Task	min Time to finish	Task	min Time to finish	Minimum time is <u>140 minutes</u>
1)	15	7)	50 (thru 5)	Crit. Path is <u>2, 5, 7, 8, 10</u>
2)	20	8)	80 (thru 7)	
3)	5	9)	125	
4)	22 (thru 2)	10)	140	
5)	25 (thru 2)			
6)	24 (thru 2)			

(1pt) (c) If you could reduce the time of any one task to zero, which task should you choose in order to shorten the required time as much as possible?

If you reduce task 8 to zero, you gain 30 minutes.

(~~The only~~ If you reduce task 10, you still have to do 9 after 8,
So you only gain 15 minutes)

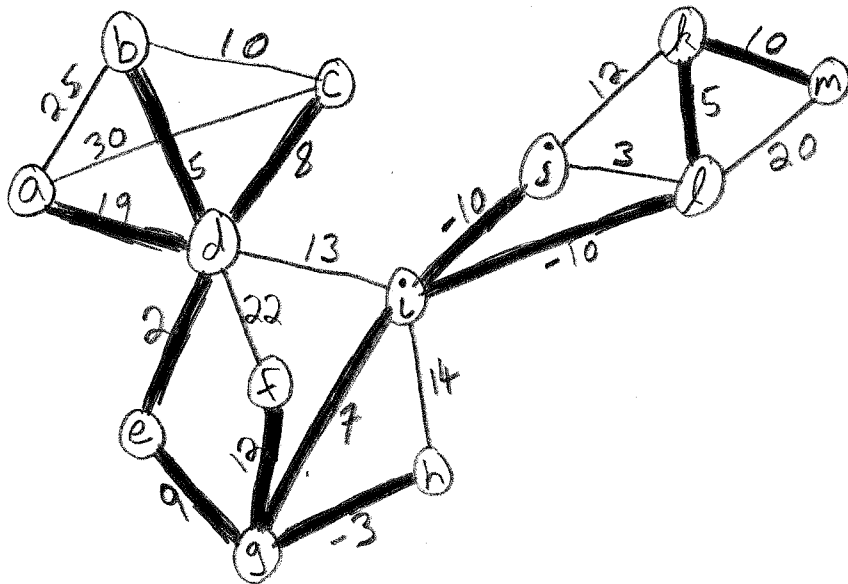
- (7pt) 6. Run Dijkstra's algorithm on the following digraph to determine the lengths of the shortest paths from s to each of the other vertices. Indicate (by highlighting or darkening) the edges that form the shortest paths tree.



	a	b	c	d	e
s	(2,s)	6,s	10,s	∞	∞
a		(6,s)	10,s	12,a	∞
b			10,s	(9,b)	∞
d			(10,s)		14,d
c					(14,d)

(2,s) (6,s) (10,s) (9,b) (14,d)

- (7pt) 7. Find a minimum spanning tree for the following graph. You may use either Prim's algorithm or Kruskal's algorithm. Please indicate which algorithm you are using and list the edges in the order they are chosen. Indicate (by highlighting or darkening) the edges that are in the tree. If you choose Prim's algorithm, start at vertex a .



<u>Prim</u>	<u>Kruskal</u>
(a,d) 19	(e,j) -10
(d,b) 5	(i,l) -10
(d,e) 2	(g,h) -3
(c,d) 8	(d,e) 2
(e,g) 9	(d,b) 5
(g,h) -3	(h,l) 5
(g,i) 7	(g,i) 7
(i,j) -10	(c,d) 8
(i,l) -10	(e,g) 9
(k,l) 5	(k,m) 10
(k,m) 10	(g,f) 12
(g,f) 12	(a,d) 19

Honor Statement:

I understand that it is the responsibility of every member of the Carolina community to uphold and maintain the University of South Carolina's Honor Code. As a Carolinian, I certify that I have neither given nor received unauthorized aid on this exam.

Signature: _____