MATH 374-001 Test 3

April 4, 2014

Name: Key

Directions: You have 50 minutes to complete the following exam. Show all applicable work. Answers without proper evidence of understanding will not receive credit.

1. Consider the following sets:

$$A = \{1, 3, \pi, 4, 9, 10\} \qquad C = \{1, 3, \pi\}$$

$$B = \{\{1\}, 3, 9, 10\} \qquad D = \{\{1, 3, \pi\}, 1\}$$

(a) (2 pts.) What is the cardinality of $A \times \mathcal{P}(D)$?

6×2^2

$= 6 \times 4 = \boxed{24}$

(b) (2 pts.) What is the cardinality of $\mathcal{P}(A \cap B)$?

$2^{(3)} = \boxed{8}$

(c) (6 pts.) Circle T if the statement is true, and F if it is false.

i.	T	F	$B \subseteq A$	vii.	T	F	$C \subseteq A$
ii.	T	F	$1 \in A$	viii.	T	F	$\{1\} \in B$
iii.	T	F	$1 \in B$	ix.	T	F	$\emptyset \subset B$
iv.	T	F	$1 \subseteq D$	x.	T	F	$C \subseteq D$
v.	T	F	$\{1\} \subseteq C$	xi.	T	F	$C \in D$
vi.	T	F	$\{1\} \subseteq B$	xii.	T	F	$C \notin A$

2. Suppose we have 5 red balls, numbered 1-5, 3 green balls numbered 1-3, and 7 white balls, numbered 1-7.

(a) (2 pts.) How many different collections of 4 balls are there? (A collection is an unordered set)

$$\binom{15}{4}$$

(b) (2 pts.) How many different 4 ball collections are there using only the white and green balls?

$$\binom{10}{4}$$

(c) (2 pts.) How many different 4 ball collections are there using at least one red ball?

$$\binom{15}{4} - \binom{10}{4}$$

(d) (2 pts.) How many different 4 ball collections are there using all three colors?

Opton 2 reds

$$\binom{5}{2} \cdot 3 \cdot 7$$

Opton 2 green

$$5 \cdot \binom{3}{2} \cdot 7$$

Opton 2 white

$$5 \cdot 3 \cdot \binom{7}{2}$$

$$\boxed{\binom{5}{2} \cdot 3 \cdot 7 + 5 \cdot \binom{3}{2} \cdot 7 + 5 \cdot 3 \cdot \binom{7}{2}.}$$

3. We have a classroom of 50 students. 20 students are male, and 30 are female. For the following problems, we want to create a committee of 10 students. The committee is composed of a chairperson, a vice chairperson, a secretary, and 7 ordinary members.

(a) (2 pts.) How many way can we form the committee?

$$P(50,3)\cdot\binom{47}{7}$$

choose chair, vice, sec.

Choose remaining 7

(b) (3 pts.)How many ways can we form the committee if it must include at least one male student?

#all female is $P(30,3)\cdot\binom{27}{7}$.

So # at least one male is

$$\boxed{P(50,3)\cdot\binom{47}{7}-P(30,3)\cdot\binom{27}{7}.}$$

(c) (3 pts.) How many ways can we form the committee if the chairperson and vice chairperson must be of opposite sexes?

Option Chair = M, VC = F.

step: Chair 20

step: V.C 30.

step Sec. 48

step Rest $\binom{47}{7}$.

$$2\left(20\cdot 30\cdot 48\cdot\binom{47}{7}\right).$$

Option Chair F, VC = M.

step Chair 30

step VC 20

step Sec 48

step Rest $\binom{47}{7}$

4. (5 pts.) How many different *10 letter* words can be made using the 11 letters in the word ABRACADABRA if each distinct letter must be used at least once. Some examples: ABRACADABR is good, since it uses 10 of the letters, and uses A, B, C, D and R at least once. ABRAADABRA is bad, since it doesn't use the letter C. AAAAAABCDR is bad, because there are only 5 A's available in ABRACADABRA.

5-A's 2B's 2R's 1C 1D.

option skip an A.

$$\frac{10!}{4!2!2!} \text{ ways}$$

option skip a B.

$$\frac{10!}{5!2!} \text{ ways.}$$

option skip R

$$\frac{10!}{5!2!} \text{ ways.}$$

ways is

$$\frac{10!}{4!2!} + \frac{10!}{5!2!} \cdot 2.$$

5. (a) (2 pts.)Determine the number of *non-negative* integer solutions are there to the equation

$$x_1 + x_2 + x_3 + x_4 + x_5 = 21.$$

Stars + Bars. 21 stars, 4 separators (for the 5 variables)

$21 + (5-1)$ items, place 4 separators

$$\binom{25}{4} \text{ ways.}$$

(b) (2 pts.) Determine the number of *positive* integer solutions are there to the equation

$$x_1 + x_2 + x_3 + x_4 + x_5 = 21.$$

give each variable 1, Then we want non negative solutions

to $X_1 + X_2 + X_3 + X_4 + X_5 = 16$. Stars + Bars.

$$\binom{20}{4} \text{ ways.}$$

6. (3 pts.)The Math department building at a certain university has 12 available classrooms. Each of these classrooms can be assigned for 7 MWF class time slots, and 5 TTh class time slots. Due to a particularly amazing instructor named Daron Tutle, interest in mathematics courses has skyrocketed, and so the department is planning to offer 150 courses next semester. Show that the department will need to hold some classes in a building other than their own.

$12 \cdot 12$ possible optons for classes.

Since $150 > 144$, Pigeon hole says

anyway we assign classes to places, two classes are in the same time & Place.

No go.

7. (3 pts.) Show that in any collection of 8 positive integers, there is a pair $\{a, b\}$ where $a - b$ is divisible by 7. (Hint: Each of the 8 integers can be written as $7q + r$ where $r = 0, 1, 2, 3, 4, 5$ or 6.)

Let #s be pigeons, Let remainder on division be

The pigeonholes.

Since $8 > 7$, two #s a, b have the same remainder

r. Hence There are $a = 7q_1 + r$, $b = 7q_2 + r$

So $a - b = 7(q_1 - q_2)$

is divisible by 7.

8. (5 pts.) Bing! is testing a new promotional tool to increase traffic to their search engine. They pay one penny to every other person who does a search, a nickel to every third person who does a search, and a dime to every fifth person who does a search. (So searcher 1 earns nothing, 2 gets a penny, 3 gets a nickel, 4 gets a penny, 5 gets a dime, 6 gets a penny *and* a nickel, 7 gets nothing, 8 gets a penny, 9 gets a nickel, 10 gets a penny and a dime, etc.) In the first year, they had 210 searches. How many searches earned at least one prize?

A = People who got penny
B = Got nickel
C = Got dime.

$A \cap B$ is div. by $\underline{6}$ − 35 of these.
$A \cap C$ div by $\underline{10}$ − 21 of these.
$B \cap C$ div by $\underline{15}$ − 14 of these.
$A \cap B \cap C$ = div by $\underline{30}$ − 7 of these.

$|A| = 210/2 = 105$

$|B| = 210/3 = 70$

$|C| = 210/5 = 42.$

$-35-21-14+7$

$|A \cup B \cup C| = 105 + 70 + 42 - \cancel{6\ 10\ 15\ +30}$

$105+35+21-7$

$140+14 = \boxed{154}$

9. (a) (2 pts.) Write the coefficient of x^4 in $(1+2x)^{12}$.

term is $i=4$.

$\binom{12}{4} 1^8 (2x)^4 = \binom{12}{4} \cdot 2^4 \cdot x^4$

(b) (2 pts.) Evaluate $\sum_{i=0}^{12} \binom{12}{i} 2^i$.

$= \sum_{i=0}^{12} \binom{12}{i} 1^{n-i} 2^i$

$= (1+2)^{12} = \boxed{3^{12}}$

Honor Statement:

I understand that it is the responsibility of every member of the Carolina community to uphold and maintain the University of South Carolina's Honor Code. As a Carolinian, I certify that I have neither given nor received unauthorized aid on this exam.

Signature: ______________________________