MATH 374-001 Test 1

February 7, 2014

Name: ____

Directions: You have 50 minutes to complete the following exam. Show all applicable work. Answers without proper evidence of understanding will not receive credit.

1. (5 pts.) Construct a truth table for the following wff:

$$[(A \wedge B) \vee (B \to A)]'.$$

A	B	$A \wedge B$	$B \to A$	$(A \wedge B) \vee (B \to A)$	$[(A \wedge B) \vee (B \to A)]'$
T	T	T	T	T	F
T	F	F	T	T	F
F	T	F	F	F	T
F	F	F	T	T	F

2. (3 pts. each) Negate each of the following statements:

(a) All people are tall and thin.

$((\forall x)[T(x) \wedge Th(x)])' \leftrightarrow (\exists x)\, T'(x) \vee Th'(x)$

"Some person is short or Fat."

(b) If there is smoke, then there is fire.

$(S \to F)' \leftrightarrow (S' \vee F)' \leftrightarrow S \wedge F'$

"There is smoke, but no fire."

3. (4 pts.) Translate the compound statement into symbolic notation using the given symbols:

If it is Monday but there is not snow is in the forecast, then we will have class.

M It is Monday.
S There is snow is in the forecast.
C We have class.

$M \wedge S' \to C$

4. (6 pts.) Prove that the argument is valid:

$$(P' \wedge Q) \wedge (S \to P) \wedge (S' \vee A \to T) \to (T \wedge Q).$$

1) $P' \wedge Q$ hyp
2) $S \to P$ hyp
3) $S' \vee A \to T$ hyp
4) P' simp (1)
5) S' mt (4, 2)
6) $S' \vee A$ add (5)
7) T mp (6, 3)
8) Q simp, (1)
9) $T \wedge Q$ conj (7, 8)

5. (5 pts.) Translate the argument to symbolic notation. Let the domain be all people, and use predicate symbols A(x), N(x), G(x), C(x). You do *not* need to prove the validity of the argument.

There is an astronomer who is not nearsighted. Everyone who wears glasses is nearsighted. Everyone wears glasses or contacts. Therefore some astronomer wears contacts.

$(\exists x)(A(x) \wedge N(x)') \wedge (\forall x)(G(x) \to N(x)) \wedge (\forall x)(G(x) \vee C(x)) \to (\exists x)(A(x) \wedge C(x))$.

6. (6 pts.) Prove the following wff is a valid argument:

$$(\forall x)[(A(x))' \vee B(x)] \to [(\exists x)A(x) \to (\exists x)B(x)].$$

By deduction, we prove instead

$(\forall x)[A(x)' \vee B(x)] \wedge (\exists x)A(x) \to (\exists x)B(x)$

1) $(\forall x)[A(x)' \vee B(x)]$ hyp
2) $(\exists x)\ A(x)$ hyp
3) $A(a)$ e.i. (2)
4) $A(a)' \vee B(a)$ u.i. (1)
5) $A(a) \to B(a)$ imp, (4)
6) $B(a)$ mp, (3,5)
7) $(\exists x) B(x)$ eg. (6)

7. (4 pts.) Show, using an interpretation, that the following wff is not valid:

$$(\forall x)P(x) \vee (\exists x)Q(x) \to (\forall x)[P(x) \vee Q(x)].$$

Let D be the positive integers.
Let $P(x)$ = "x is divisible by 4"
Let $Q(x)$ = "x is divisible by 2."

The left side of the implication says
" Either all integers are divisible by 4, or there is some integer divisible by 2."
This is true, because (for example) 4 is divisible by two.
The right side says "For every integer, it is divisible by 4 or by 2."
This is false, because 3 is not divisible by either.

Hence the implication is false.

8. (4 pts.) Verify the correctness of the following program segment used to compute $y = x(x+1)/2$. (Use $\{y = x(x+1)/2\}$ as the postcondition.)

$y = x * x$
$y = y + x$
$y = y/2.$

$$\left\{ \frac{x*x+x}{2} = \frac{x(x+1)}{2} \right\} \longleftrightarrow \left\{ \frac{x(x+1)}{2} = \frac{x(x+1)}{2} \right\}$$

$y = x*x$

$$\left\{ \frac{y+x}{2} = x(x+1)/2 \right\}$$

$y = y+x$

$$\left\{ y/2 = x(x+1)/2 \right\}$$

$y = y/2$

$$\left\{ y = x(x+1)/2 \right\}$$

These are all true by the assignment rule. Since the precondition is obviously true, the program is correct.

9. (5 pts.) Verify the correctness of the following program segment with the given precondition and postcondition.

$\{x \neq 0\}$
  if $x > 0$ **then**
    $y = 2 * x$
  else
    $y = (-3) * x$
  end if
$\{y > 0\}$

We prove 2 statements.

1) $\{x \neq 0 \wedge x > 0\}$ $y = 2*x$ $\{y > 0\}$.

By assignment, we have:

$\{2*x > 0\}$
$y = 2*x$ is true.
$\{y > 0\}$

But $\{2x > 0\} \Updownarrow \{x > 0\} \Updownarrow \{x \neq 0 \wedge x > 0\}$.

So statement 1) holds.

2) $\{x \neq 0 \wedge x \leq 0\}$ $y = (-3)*x$ $\{y > 0\}$.

By assignment, we know

$\{(-3)*x > 0\}$
$y = (-3)*x$ is true.
$\{y > 0\}$

But $\{-3x > 0\} \Updownarrow \{0 > 3x\} \Updownarrow \{x \neq 0 \wedge x \leq 0\}$.

So 2) holds also.

By the conditional rule, the program is correct.

Honor Statement:

I understand that it is the responsibility of every member of the Carolina community to uphold and maintain the University of South Carolina's Honor Code. As a Carolinian, I certify that I have neither given nor received unauthorized aid on this exam.

Signature: ____________________