Math 374

Equivalence and Inference Rules

TABLE 1. Equivalence Rules

Expression	Equivalent to	Name/Abbreviation
$P \vee Q$	$Q \vee P$	Commutative / comm
$P \wedge Q$	$Q \wedge P$	
$(P \vee Q) \vee R$	$P \vee (Q \vee R)$	Associative / ass
$(P \wedge Q) \wedge R$	$P \wedge (Q \wedge R)$	
$P \vee (Q \wedge R)$	$(P \vee Q) \wedge (P \vee R)$	Distributive / dist
$P \wedge (Q \vee R)$	$(P \wedge Q) \vee (P \wedge R)$	
$(P \wedge Q)'$	$P' \vee Q'$	De Morgan's laws / DeMorg
$(P \vee Q)'$	$P' \wedge Q'$	
$P \to Q$	$P' \vee Q$	Implication / imp
P	$(P')'$	Double negation / dn
$P \leftrightarrow Q$	$(P \to Q) \wedge (Q \to P)$	Equivalence / equ

TABLE 2. Inference Rules

From	Can Derive	Name/Abbreviation
$P, P \to Q$	Q	Modus Ponens / mp
$P \to Q, Q'$	P'	Modus Tollens / mt
P, Q	$P \wedge Q$	Conjunction / conj
$P \wedge Q$	P, Q	Simplification / simp
P	$P \vee Q$	Addition / add

TABLE 3. Predicate Logic Inference Rules

From	Can Derive	Name/Abbreviation	Restrictions
$(\forall x)P(x)$	$P(t)$ where t is a variable or constant.	Universal Instantiation/ui	If t is a variable, it must not be quantified inside $P(x)$.
$(\exists x)P(x)$	$P(a)$ where a is a constant.	Existential Instantiation/ei	Must be the first use of the constant a
$P(x)$	$(\forall x)P(x)$	Universal Generalization/ug	$P(x)$ hasn't been deduced from hypotheses where x is free, nor by using ei on a wff with x free.
$P(x)$ or $P(a)$	$(\exists x)P(x)$	Existential Generalization/eg	To get $(\exists x)P(x)$ from $P(a)$, x can't already appear in $P(a)$.