Math 374 Name:________________________________

Quiz 6

(1) A bank has 214 customers. 189 of them have checking accounts, 73 have normal savings accounts, and 114 have money market savings accounts. 69 customers have checking accounts and normal savings accounts. No customer is allowed to have a normal savings account and a money market savings account.

(a) How many customers have checking accounts and money market savings accounts?

A = Checking B = Normal Savings C = MM Savings.

$|A \cup B \cup C| = 214$. $|A| = 189$ $|B| = 73$ $|C| = 114$.

$|A \cap B| = 69$. $|B \cap C| = 0$. $|A \cap B \cap C| = 0$. By I-E.

$214 = 189 + 73 + 114 - 69 - |A \cap C| - 0 + 0 \rightarrow |A \cap C| = \boxed{93}$

(b) How many customers have a checking account, but no savings account at all?

$|A| - |A \cap C| - |A \cap B| + |A \cap B \cap C|$

$189 - 93 - 69 + 0$

$= \boxed{27}$

(2) How many numbers must be selected from the set $\{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$ to guarantee the at least one pair adds up to 22?

Only The pairs (2,20), (4,18), (6,16), (8,14), (10,12) add to 22.

If you select one from each pair, you get 5 numbers and no pair of Them add to 22.

If you select for some pair 6, you must select 2 numbers from The same pair, by The pigeonhole principle.

Hence 6 numbers guarantee a pair adding to 22.

(3) The following two problems concern a square grid of size n. Think of it as being in the first quadrant of the plane, and the intersection points are at integer paired points.

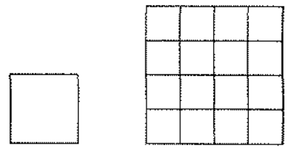

FIGURE 1. A 1×1 grid, and a 4×4 grid.

A particle starts at the bottom left corner (0,0), and travels to the top right corner (n, n). It can only make moves of exactly one unit to the right or up, and cannot leave the grid.

(a) How many different paths can the particle take in an $n \times n$ grid? (Hint: How many steps does it take?)

You must make $2n$ moves to get to (n,n) (n moves right, n moves up.). You can specify a path by telling which moves were <u>up</u> (The rest <u>must</u> be right). hence There are $\binom{2n}{n}$ paths.

(b) How many paths can the particle take that pass through the point $(4, 4)$ in a 10×10 grid?

To make a path, you must go to (4,4) Then go to (10,10). The grid from (4,4) to (10,10) is just a 6x6 grid. so There are

$\binom{8}{4} \cdot \binom{12}{6}$ paths.

which path to (4,4)

rest of path to (10,10)

(4) There are 6 people in a wedding party. How many ways can they be lined up, if the bride and groom are not allowed to stand next to each other?

$6!$ = # of ways to order the party.

$5!$ with ~~Bride left~~, Bride exactly left of Groom

$5!$ with Bride exactly <u>right</u> of groom.

So $6! - 2\cdot(5!)$ ways with Bride and Groom separated.

(5) What is the exponent of $x^5y^3z^2$ in $(x - y + 2z)^{10}$? (Hint: Use the binomial theorem twice, the first time with $x - y$ as a single term.)

$(a+2z)^{10}$. coefficient of a^8z^2 is

$\underline{\binom{10}{2}\cdot 2^2}$ by binomial Theorem.

if $a = x-y$, Then the coefficient of

x^5y^3 in $a^8 = (x-y)^8$ is

$-\binom{8}{3}$ by the binomial Theorem. So

The coefficient of $x^5y^3z^2$ is

$$\boxed{-\binom{8}{3}\binom{10}{2}\cdot 4.}$$