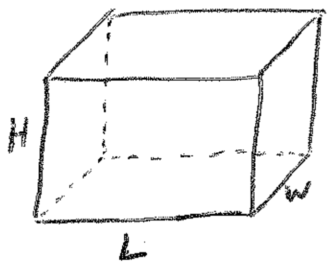
1) a



$L = 2W$, and $L \cdot W \cdot H = 10$

So $H = \frac{5}{W^2}$

$$\text{Cost} = \underset{\text{Bottom}}{10(L\cdot W)} + \underset{\text{Top}}{6(L\cdot W)} + \underset{\text{2 sides}}{6(2HW)} + \underset{\text{other 2 sides}}{6(2\cdot LH)}$$

$$C(w) = 32w^2 + \frac{60}{w} + \frac{120}{w} = 32w^2 + \frac{180}{w}.$$

Domain is $w > 0$.

<u>Minimize</u> $C(w) = 32w^2 + \frac{180}{w}$ on $(0, \infty)$.

$C'(w) = 64w - \frac{180}{w^2}$. This is zero when

$$64w = 180/w^2 \qquad w^3 = \frac{45}{16} \qquad w = \sqrt[3]{\frac{45}{16}}$$

also, $C'(w)$ is negative on $\left(0, \sqrt[3]{\frac{45}{16}}\right)$

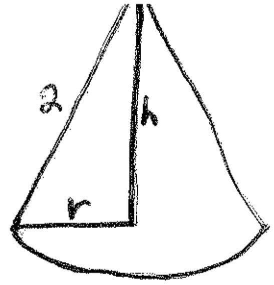
Positive on $\left(\sqrt[3]{\frac{45}{16}}, \infty\right)$

So $C\left(\sqrt[3]{45/16}\right) = 32\cdot\left(\sqrt[3]{45/16}\right)^2 + \frac{180}{\sqrt[3]{45/16}}$

≈ 191.28 \$

is the minimum Cost

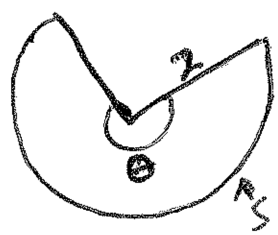
2)b



$V = \frac{1}{3}\pi r^2 h.$

in The cone

$s = 2\pi r.$



$s = 2\theta$

So $r = \frac{\theta}{\pi}$

and so $h = \sqrt{4 - \frac{\theta^2}{\pi^2}}$

So $V = \frac{1}{3}\pi \frac{\theta^2}{\pi^2}\cdot\left(4 - \frac{\theta^2}{\pi^2}\right)^{1/2}$

$= \frac{1}{3\pi^2}\theta^2\left(4\pi^2 - \theta^2\right)^{1/2}$

$0 \le \theta \le 2\pi.$

$V'(\theta) = \frac{1}{3\pi^2}\left[2\theta\left(4\pi^2-\theta^2\right)^{1/2} + \theta^2\left(\frac{1}{2}\cdot\left(4\pi^2-\theta^2\right)^{-1/2}\cdot(-2\theta)\right)\right.$

$= \frac{\theta}{3\pi^2}\left[\frac{8\pi^2 - 2\theta^2 - \theta^2}{\sqrt{4\pi^2-\theta^2}}\right]$

$= 0$ when $\theta = 0$ or $3\theta^2 = 8\pi^2$

$\theta^2 = \frac{8\pi^2}{3}$

$\theta = \frac{2\sqrt{2}\pi}{3}$

$V\left(\frac{2\sqrt{2}\pi}{3}\right) = \frac{1}{3\pi^2}\cdot\left(\frac{8\pi^2}{3}\right)\left(4\pi^2 - \frac{8\pi^2}{3}\right)^{1/2}$

$= \frac{16\pi}{9\sqrt{3}}.$ (check $\theta = 0$, $\theta = 2\pi$, Both have $V = 0$.)

Max is

2) a) $f(x) = x - 3x^2 + C.$

$8 = f(0) = 0 - 3(0)^2 + C, \quad$ so $C = 8.$

$\underline{f(x) = x - 3x^2 + 8}$

b) $g'(x) = -\cos(x) + \sin(x) + C$

$g'(0) = -1 + 0 + C = 4,$

so $C = 5.$

$g'(x) = -\cos(x) + \sin(x) + 5.$

$g(x) = -\sin(x) - \cos(x) + 5x + B$

$g(0) = -0 - 1 + 5 \cdot (0) + B = 3$

so $B = 4.$

$\underline{g(x) = -\sin(x) - \cos(x) + 5x + 4}$

3) a) $\Delta x = 1, \quad x_1 = 1, \quad x_2 = 2 \quad x_3 = 3. \quad f(x_1) = 9, \ f(x_2) = 9, \ f(x_3) = 5$

$R_3 = 1(9 + 9 + 5) = \underline{23}$

b) $\Delta x = 1 \quad x_1 = .5 \quad x_2 = 1.5 \quad x_3 = 2.5$

$f(x_1) = 6 \quad f(x_2) = 10 \quad f(x_3) = 6$

$M_3 = 1(6 + 10 + 6) = \underline{22}$

c) $\Delta x = 1/2. \quad x_1 = .5, \ x_2 = 1, \ x_3 = 1.5, \ x_4 = 1.5 \quad x_5 = 2 \quad x_6 = 2.5$

$f(x_1) = 6 \ f(x_2) = 9 \ f(x_3) = 10 \ f(x_4) = 10 \ f(x_5) = 9 \ f(x_6) = 6.$

$U_6 = .5(6 + 9 + 10 + 10 + 9 + 6) = \underline{25}$

3) d) $\Delta x = .5$ $x_1 = 0$ $x_2 = .5$ $x_3 = 1$ $x_4 = 1.5$ $x_5 = 2$ $x_6 = 2.5$

$f(x_1) = 1$ $f(x_2) = 6$ $f(x_3) = 9$ $f(x_4) = 10$ $f(x_5) = 9$ $f(x_6) = 6$

$L_6 = .5(1+6+9+10+9+6) = \underline{20.5}$

e) $\Delta x = 1.5$ $x_1 = 1.5$ $x_2 = 3$

$f(x_1) = 10$ $f(x_2) = 5$

$R_2 = 1.5(10+5) = \underline{22.5}$

---

4) a) $\Delta x = 1$ $x_1 = 2$, $x_2 = 3$ $x_3 = 4$, $x_4 = 5$

$f(x_1) = 6$ $f(x_2) = 12$ $f(x_3) = 20$ $f(x_4) = 30$

$R_4 = 1(6+12+20+30) = \underline{68}$

b) $\Delta x = 1$ $x_1 = 1.5$ $x_2 = 2.5$ $x_3 = 3.5$ $x_4 = 4.5$

$f(x_1) = 3.75$ $f(x_2) = 8.75$ $f(x_3) = 15.75$ $f(x_4) = 24.75$

$M_4 = 1(3.75 + 8.75 + 15.75 + 24.75) = \underline{53}$

c) $\Delta x = .5$ $x_1 = 1.5$ $x_2 = 2$ $x_3 = 2.5$ $x_4 = 3$ $x_5 = 3.5$ $x_6 = 4$ $x_7 = 4.5$ $x_8 = 5$

$f(x_i)$ (are written in parts a and b).

$R_8 = .5(3.75 + 6 + 8.75 + 12 + 15.75 + 20 + 24.75 + 30) = \underline{60.5}$

d) $\Delta x = \frac{4}{n}$ $x_i = 1 + i \cdot \frac{4}{n}$ $f(x_i) = \left(1+\frac{4i}{n}\right)^2 + \left(1+\frac{4i}{n}\right)$

So $\displaystyle\int_1^5 x^2 + x\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{4}{n}\left(\left(1+\frac{4i}{n}\right)^2 + \left(1+\frac{4i}{n}\right)\right)$.

5) a) $\Delta x = 4$ $x_1 = 4$ $x_2 = 8$ $x_3 = 12$ $x_4 = 16$.

$$R_4 = 4(e^4 + e^8 + e^{12} + e^{16})$$

b) $\Delta x = 4$ $x_1 = 2$ $x_2 = 6$ $x_3 = 10$ $x_4 = 14$

$$M_4 = 4(e^2 + e^6 + e^{10} + e^{14})$$

c) $\Delta x = 2$ $x_1 = 2$ $x_2 = 4$ $x_3 = 6$ $x_4 = 8$ $x_5 = 10$ $x_6 = 12$ $x_7 = 14$ $x_8 = 16$

$$R_8 = 2(e^2 + e^4 + e^6 + e^8 + e^{10} + e^{12} + e^{14} + e^{16})$$

d) $\Delta x = \frac{16}{n}$ $x_i = 0 + \frac{16i}{n}$ $f(x_i) = e^{\frac{16i}{n}}$

so $$\int_0^{16} e^x dx = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{16}{n} e^{\frac{16i}{n}}$$

6 a) $= 2r e^{r^2}$

b) $= \frac{d}{dx}\left[-\int_5^{x^3} 2\cos t \sin(t)\, dt \right]$

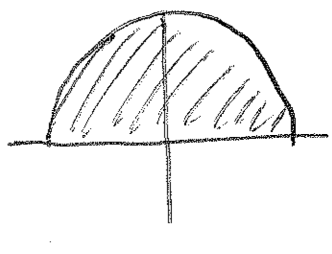
$= -2\cos(x^3)\sin(x^3) \cdot 3x^2$

c) $= (10^3 + 25\cdot 10) - (2^3 + 25\cdot 2)$

$= 1250 - 58$

$= \underline{1192}$

d) $\underline{= 0}$

e) 

area of half circle is (radius 2)

$$\frac{\pi (2)^2}{2} = \boxed{2\pi}$$

7) a) $\int x^2 - x^{-2}\,dx = \dfrac{x^3}{3} + \dfrac{1}{3x^3} + C$

b) $\int \csc^2(t) - 2e^t\,dt = -\cot(t) - 2e^t + C$

c) $\int \sec(x)\left(\sec(x) + \tan(x)\right)dx = \int \sec^2(x) + \sec(x)\tan(x)\,dx$

$= \tan(x) + \sec(x) + C$

d) $\int x^2 + 1 + \dfrac{1}{x^2+1}\,dx = \dfrac{x^3}{3} + x + \arctan(x) + C$

e) $\int \dfrac{x^3 - 2\sqrt{x}}{x}\,dx = \int x^2 - 2x^{-1/2}\,dx$

$= \dfrac{x^3}{3} - 4x^{1/2} + C$

f) $\int (3x-2)^{20}\,dx = \frac{1}{3}\int (3x-2)^{20}\,3dx$

$u = 3x - 2$

$du = 3\,dx$

$= \frac{1}{3}\int u^{20}\,du$

$= \dfrac{u^{21}}{63} + C = \dfrac{(3x-2)^{21}}{63} + C$

g) $\int \dfrac{x}{(x^2+1)^2}\,dx = \frac{1}{2}\int \dfrac{1}{(x^2+1)^2}\,(2x\,dx)$

$u = x^2 + 1$

$du = 2x\,dx$

$= \frac{1}{2}\int \frac{1}{u}\,du$

$= \frac{1}{2}\ln(u) + C = \frac{1}{2}\ln(x^2+1) + C$

7) h) $\int e^x \sin(e^x)dx = \int \sin(u)\,du$

$u = e^x$

$du = e^x dx$

$= -\cos(u) + C$

$= \boxed{-\cos(e^x) + C}$

i) $\int e^{\tan(x)} \sec^2(x)dx = \int e^u\,du$

$u = \tan(x)$

$du = \sec^2(x)dx$

$= e^u + C$

$= \boxed{e^{\tan(x)} + C}$

j) $\int \frac{e^x}{e^x+1}dx = \int \frac{1}{u}\,du$

$u = e^x+1$

$du = e^x dx$

$= \ln(u) + C$

$= \boxed{\ln(e^x+1) + C}$

---

8) a) $\int_1^2 \frac{3}{t^4}dt = \int_1^2 3t^{-4}dt = \left.\frac{3t^{-3}}{-3}\right|_1^2$

$= \left.-\frac{1}{t^3}\right|_1^2 = \left(-\frac{1}{2^3}\right) - \left(-\frac{1}{1^3}\right)$

$= \boxed{\frac{7}{8}}$

8) b) $\int_{\pi}^{2\pi} \cos(x)\,dx = \sin(x)\Big|_{\pi}^{2\pi} = 0 - 0 = \boxed{0}$

c) $\int_1^9 \frac{1}{2x}\,dx = \frac{1}{2}\int_1^9 \frac{1}{x}\,dx = \frac{1}{2}\left(\ln(x)\Big|_1^9\right)$

$= \frac{1}{2}\left(\ln(9) - \ln(1)\right)$

$= \boxed{\ln(3)}$

---

d) $\int_1^2 (1+2y)^2\,dy = \frac{1}{2}\int_3^5 u^2\,du$

$u = 1+2y$

$du = 2dy$ $\qquad = \frac{1}{2}\left(\frac{u^3}{3}\Big|_3^5\right)$

$1 \to 3$

$2 \to 5$ $\qquad = \frac{1}{2}\left(\frac{125}{3} - \frac{27}{3}\right) = \boxed{\frac{49}{3}}$

---

e) $\int_0^{\sqrt{\pi}} x\cos(x^2)\,dx = \frac{1}{2}\int_0^{\sqrt{\pi}} \cos(x^2)(2x\,dx)$

$u = x^2 \quad du = 2x\,dx$

$x=0 \to u=0$

$x=\sqrt{\pi} \to u=\pi$.

$= \frac{1}{2}\int_0^{\pi} \cos(u)\,du = \frac{1}{2}\left[\sin(u)\Big|_0^{\pi}\right]$

$= \frac{1}{2}\left[0 - 0\right] = 0.$

f) $\int_0^2 (x-1)^{25}\, dx$

$u = x-1 \qquad x=0 \rightarrow u=-1$

$du = dx \qquad x=2 \rightarrow u=1$

$$= \int_{-1}^{1} u^{25}\, du = \frac{u^{26}}{26}\Big|_{-1}^{1} = \left(\frac{1^{26}}{26}\right) - \left(\frac{(-1)^{26}}{26}\right)$$

$$= \boxed{0}$$

g) $\int_0^1 x e^{-x^2} dx = -\frac{1}{2}\int_0^1 e^{-x^2}(-2x\,dx)$

$u = -x^2 \qquad x=0 \rightarrow u=0$

$du = -2x\,dx \qquad x=1 \rightarrow u=-1.$

$$= -\frac{1}{2}\int_0^{-1} e^u\, du = \frac{1}{2}\int_0^1 e^u\, du$$

$$= \frac{1}{2} e^u\Big|_0^1 = \boxed{\frac{1}{2}e - \frac{1}{2}}$$

h) $\int_0^{13} \frac{dx}{\sqrt[3]{(1+2x)^2}} = \frac{1}{2}\int_0^{13} \frac{1}{(1+2x)^{2/3}}(2\,dx)$

$u = 1+2x \qquad x=0 \rightarrow u=1$

$du = 2dx \qquad x=13 \rightarrow u=27$

$$= \frac{1}{2}\int_1^{27} u^{-2/3}\, du = \frac{1}{2}\left[3u^{1/3}\Big|_1^{27}\right]$$

$$= \frac{1}{2}\left[3\cdot 27^{1/3} - 3\cdot 1^{1/3}\right]$$

$$= \frac{1}{2}[9-3] = \boxed{3}$$

9) a) $\int_{60}^{120} r(t)\,dt$ is how much water leaked out over the second hour.

b) $\int_{5}^{10} w'(t)\,dt$ is the net weight gain from years 5 to 10.

c) $\int_{4}^{10} 200+5t\,dt$

$$= 200t + \frac{5}{2}t^2 \Big|_4^{10}$$

$$= \left[(2000+250)-(800+40)\right]$$

$$= \boxed{1410 \text{ members}}$$