

DISCRETE OPTIMIZATION: PROBLEM SET 12

For the following problems, feel free to research the solutions if you wish. Most are a bit too difficult to come up with new solutions to. Being able to explain clearly is the important aspect.

Problem 1. Present a polynomial algorithm for solving 2-SAT.

Problem 2. The problem CLIQUE takes a graph G and a number m , and asks if there is a set of m vertices that are each pairwise connected (i.e., these m vertices are a complete graph inside G). The two steps below show that CLIQUE is NP-complete.

- Explain why CLIQUE is in NP.
- Show that SATISFIABILITY reduces in polynomial time to CLIQUE. (i.e., if you could solve CLIQUE, you could solve SATISFIABILITY.)

Problem 3. Show that SATISFIABILITY reduces to 3-SAT.

Problem 4. Choose any pair of NP-complete problems, introduce them, and show a reduction from one to the other. (Wikipedia, or *Computers and Intractability* by Garey and Johnson are excellent resources for this.)

Problem 5. Present the Klee and Minty example that shows that the simplex method can have exponential runtime.

Problem 6. Present the ellipsoid algorithm, or Karmakar's algorithm, both of which solve linear optimization problems in polynomial time.