# Math 746 — Commutative Algebra — Spring 2021

## Instructor: Alexander Duncan

Commutative algebra is the study of commutative rings, which are ubiquitous in all areas of mathematics. Indeed, it is a formidable challenge to solve problems without using integers or polynomials. This course arms you with the basic concepts and techniques of the subject. This will serve as a foundation for both its applications in other areas of mathematics and for deeper study of the subject itself.

The subject's history is intimately involved with number theory — many of the bedrock notions such as ideals, integral extensions, valuations, and completions were first developed to study primes and the integers. The 20th century saw algebraic geometry completely rewritten from the ground up with commutative algebra its *lingua franca*. The subject revolutionized geometry and topology, which returned the favor with new tools like homology theory. Now, the burgeoning field of algebraic combinatorics studies the relationship of combinatorial objects like matroids and polytopes with algebraic objects like monomial ideals.

$$\cdots \to F_4 \xrightarrow{\partial} F_3 \xrightarrow{\partial} F_2 \xrightarrow{\partial} F_1 \xrightarrow{\partial} F_0 \to 0$$

The textbook is *A Term of Commutative Algebra* by A. Altman and S. Kleiman, which is available for free online. This text is a modern take on the venerable classic *Atiyah & Macdonald*. Despite its name, I do not expect to cover all the material in the text in one semester. However, topics we will see include:

- rings and modules
- homological algebra
- limits
- tensor products
- localization
- integral extensions
- primary decomposition
- valuation rings

The prerequisite for the course is Math 701. You should have at least seen rings, ideals, and modules before, but you are not expected to be fluent (yet). The course is especially synergistic with 702 (Algebra II), 737 (Introduction to Complex Geometry), 748 (Computational Algebra), and 784 (Algebraic Number Theory). Your grade will be determined by regular assignments.

$$\begin{tikzcd}
& \widehat{R} \otimes_R A \arrow[r] \arrow[d] & \widehat{R} \otimes_R B \arrow[r] \arrow[d] & \widehat{R} \otimes_R C \arrow[r] \arrow[d] & 0 \\
0 \arrow[r] & \widehat{A} \arrow[r] & \widehat{B} \arrow[r] & \widehat{C} \arrow[r] & 0
\end{tikzcd}$$