The final exam will be a take-home exam. It will be available on the course website by **May 1 at 11:30 am**. It will be due by **May 8 at 11:30 am** (you can either hand this in to me in person or by putting it in my mailbox by the deadline).

The rules are different from both the midterm and the problem sets:

- You are **allowed** to use your notes from class.
- You are **allowed** to use resources posted on the course website.
- You are **allowed** to use the course textbook.
- You are **allowed** to use Macaulay2.
- You **cannot** discuss the exam with your classmates or any other person.
- You **cannot** use other textbooks or online resources (e.g. Chegg, MathOverflow) to help solve problems.

I will be answering emails throughout the exam period. If you have any confusion or concerns about the rules above, then send me an email. I am willing to help troubleshoot technological problems with M2, but not mathematical ones.

You have a week to write the exam: do not wait until the last minute! **If you rely on an online version of M2 and it goes down during the exam period, then this is not an excuse.** If this happens, get in touch with me as soon as possible.

The final exam is cumulative. It will cover the same material from the midterms, as well as material we covered from Chapter 4. Specifically, the sections

$$\S 1.1\text{–}1.5,\ 2.1\text{–}2.8,\ 3.1\text{–}3.4,\ 4.1\text{–}4.6,\ 4.8\text{–}4.9$$

will be tested.

As with the midterms, the emphasis will be on computational skills, but there will also be some questions demanding a greater understanding of the material. Some questions will require computer algebra systems, and others will require that you write out all of the details by hand.

Note that we only learned how to find radicals of *principal ideals* by hand. Macaulay2 can find radicals of any ideal and can compute primary decompositions, which we never learned how to do in class.

Please see the previous midterm information documents to review chapters 1–3.

For chapter 4, there is suggested practice on the back of this page.

Problem 1 Consider the ideals $I = \langle x^3y\rangle$, $J = \langle xy^2\rangle$, and $K = \langle x, y\rangle$ in $\mathbb{C}[x, y]$. Find explicit descriptions of the following:

(a) $I + J$
(b) IK
(c) K^2
(d) $I \cap J$
(e) $\gcd(x^3y, xy^2)$
(f) $I : J$
(g) $I : J^\infty$
(h) $\sqrt{I}$

You probably see how to compute some of these by inspection, but make sure you can apply by hand the general algorithms from class. (For example $I \cap J$ requires a Gröbner basis calculation in general.)

Problem 2 Look at the dictionary in §4.9 of the text. Which aspects of the dictionary break down when $k = \mathbb{R}$ or the ideals are not radical? Can you find explicit counterexamples?

Problem 3 For each ideal I in $\mathbb{C}[x, y, z]$, find the minimal primary decomposition of I and determine the minimal decomposition of $\mathbf{V}(I)$ into irreducible varieties.

(a) $I = \langle x^3 - x\rangle$
(b) $I = \langle x^2y,\ xz^2\rangle$
(c) $I = \langle yz^2,\ y^2z,\ xz^2,\ xyz,\ xy^2,\ x^2z,\ x^2y\rangle$
(d) $I = \langle xz+yz+z^2-z,\ xy^2-xy+y^3+y^2z-2y^2-yz+y,\ x^2-x-y^2-2yz+y-z^2+z\rangle$

Problem 4 Find all the singular points of

$$\mathbf{V}\left(x^6 + y^6 + 1 + (x^2 + y^2 + 1)(x^4 + y^4 + 1) - 12x^2y^2\right) .$$

(Hint: Elimination theory could be used in principle, but you'll probably want to use primary decomposition and radical ideals in practice.)