Numerical Integration: The Trapezoidal Rule and Simpson's Rule

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Overview

As we have learned in Calculus I, there are two ways to evaluate a definite integral: using the Fundamental Theorem of calculus or numerical approximations. While FTC is nice in theory, it cannot be applied in many cases, as antiderivatives are often difficult or even impossible to find. Using computers, numerical integration is the most general and practical way to evaluate definite integrals.

Maple Essentials

The *Trapezoid Rule*, *Simpson's Rule*, *Trapezoid Rule Error Formula*, and *Simpson's Rule Error Formula* Maplets are useful tools to help you practice numerical integration and error approximation using the Trapezoidal Rule and Simpson's Formula. They are also useful to check any Trapezoidal Rule or Simpson's Rule approximation that you compute by hand. These maplets are available on the course website:

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`http://people.math.sc.edu/calclab/142L-S17/labs/`

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New Maple commands introduced in this lab include:

Command	Description
`ApproximateInt`	approximates a definite integral using one of the approximation methods: midpoint, trapezoid or simpson. `ApproximateInt(f(x), x=a..b, method=method, partition=n, partitiontype=normal);` Note: The `ApproximateInt` command is in the `Student[Calculus1]` package.
`maximize`	determines the maximum value of an expression over the given interval. `maximize(f(x), x=a..b);` Note: The `maximize` command is in the `Optimization` package.

Related course material/Preparation

§8.7 of Thomas' Calculus.

Geometrically, as shown in §8.7 of the text, the trapezoidal approximation and Simpson's rule use a line and a parabola (polynomials of degree 1 and 2), respectively, to approximate $y = f(x)$ in each subinterval. In the trapezoidal approximation, the line is determined by the left and right endpoints of the curve in each subinterval. To determine a parabola, you need three points. Therefore, the left endpoint, the right endpoint, and the midpoint of the curve in each subinterval are used in Simpson's rule.

Assignment

Review Labs A-E for **Hour Quiz I** next week.

Activities

1. Estimate the integral with $n = 4$ steps and approximate the integral by (a) the Trapezoidal Rule and (b) Simpson's Rule. In each case, use the exact value of the integral (as determined by Maple) to calculate the absolute error. Express your answer to at least 4 decimal places.

   (i) $\displaystyle\int_0^2 (t^3 + t)dt$ (ii) $\displaystyle\int_1^2 \frac{1}{s^2} ds$ (iii) $\displaystyle\int_2^4 \frac{1}{(s-1)^2} ds$

   (iv) $\displaystyle\int_0^\pi \sin(t)dt$ (v) $\displaystyle\int_0^3 \sqrt{x+1}dx$ (vi) $\displaystyle\int_0^3 \frac{1}{\sqrt{x+1}}$

   Note: These problems appear on page 502 in Thomas' Calculus.

2. Use (a) the Trapezoid Rule Maplet and (b) the Simpson's Rule Maplet to redo problems (i)-(vi) above.

3. Use (a) the Trapezoid Rule Error Formula Maplet and (b) the Simpson's Rule Error Formula Maplet to practice finding bounds for the error when using each method.

4. Estimate the minimum number of subintervals needed to approximate integrals (i)-(vi) above with an error of magnitude less than 10^{-4} by (a) the Trapezoidal Rule and (b) Simpson's Rule. Verify your answer by calculating the approximation in Maple.

Example: Activities 1(i) and 4(i)

- Start by including the `Student[Calculus1]` and `Optimization` packages.
  >with(Student[Calculus1]):
  >with(Optimization):

- We define our integrand and bounds so we do not have to type as much later.
  >f:= t − > t^3+t;
  >a:=0; b:=2;

- For (a) we find the trapezoidal approximation. The `evalf` command returns a decimal approximation.
  >Trap:=evalf(ApproximateInt(f(t), t=a..b, method=trapezoid, partition=4, partitiontype=normal));

- We then find the absolute error associated with this approximation.
  >ErrTrap:=evalf(abs(int(f(t), t=a..b) - Trap));

- For (b) we find the Simpson's Rule approximation.
  >Simp:=evalf(ApproximateInt(f(t), t=a..b, method=simpson, partition=4, partitiontype=normal));

- We then find the absolute error associated with this approximation.
  >ErrSimp:=evalf(abs(int(f(t), t=a..b) - Simp));

- For the Trapezoid Rule Error Formula, we need to find M such that $M \geq |f''|$ on $[0,2]$, so we will find $f''(t)$.
  >diff(f(t), t$2);
  >M:=maximize(abs(*Insert Label*), t=a..b);

- We then use the error formula $|E_T| \leq \frac{M(b-a)^3}{12n^2}$ to solve for n.
  >solve(M*(b-a)^3/(12*n^2)<10^(-4),{n});

- For the Simpson's Rule Error Formula, we need to find M such that $M \geq |f^{(4)}|$ on $[0,2]$, so we will find $f^{(4)}(t)$ and redefine M.
  >diff(f(t), t$4);
  >M:=maximize(abs(*Insert Label*), t=a..b);

- We then use the error formula $|E_S| \leq \frac{M(b-a)^5}{180n^4}$ to solve for n.
  >solve(M*(b-a)^5/(180*n^4)<10^(-4),{n});

- You can verify your answers by recalculating the approximations and associated error values.