Parallel Quality Meshes for Earth Models

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04 October 2016,
Virginia Tech

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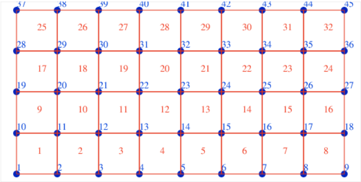
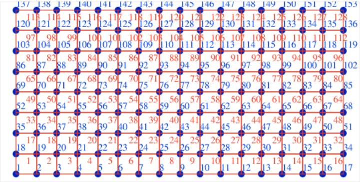
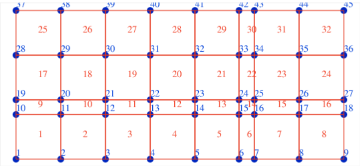
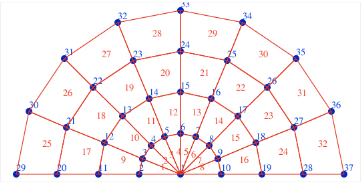
http://people.sc.fsu.edu/∼jburkardt/presentations/. . .
. . . sphere_grid_2016_vt.pdf

Meshing: Rectangular Meshing

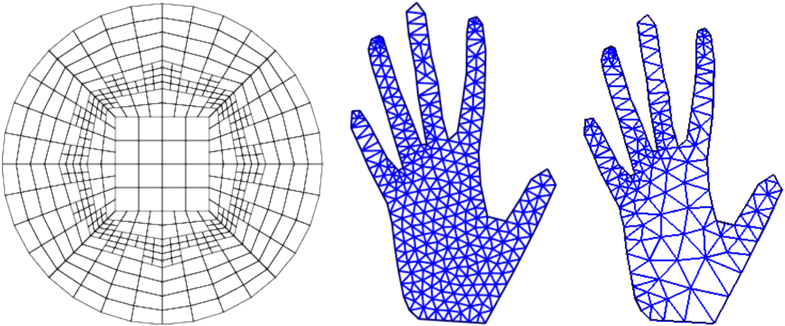




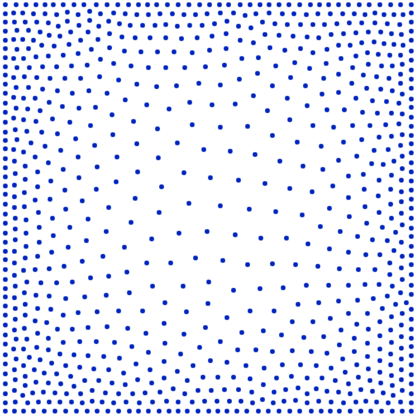


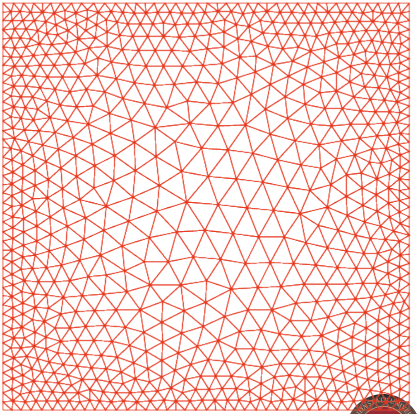


Meshing: Nested and Unstructured Grids

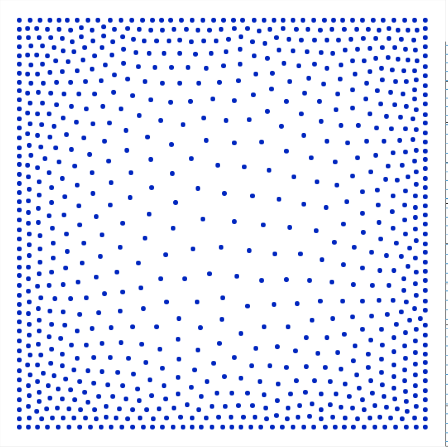


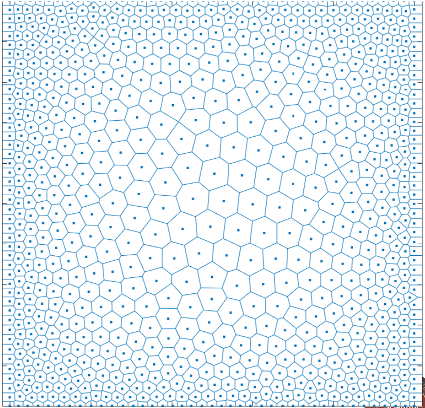
Meshing: Points and Delaunay Triangles



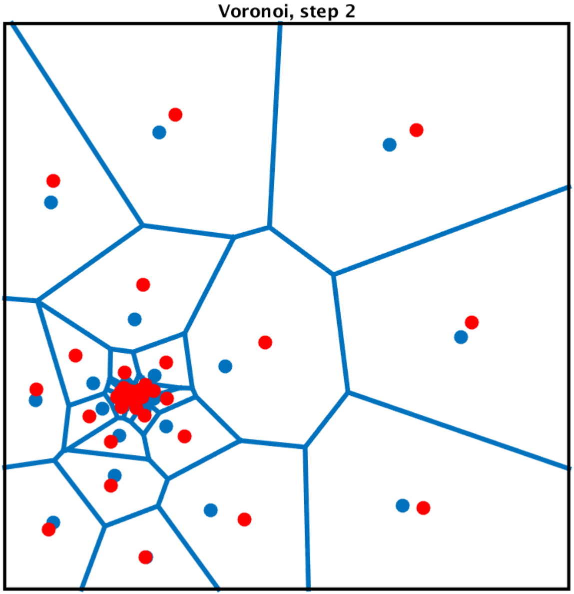


Meshing: Points and Voronoi Polygons

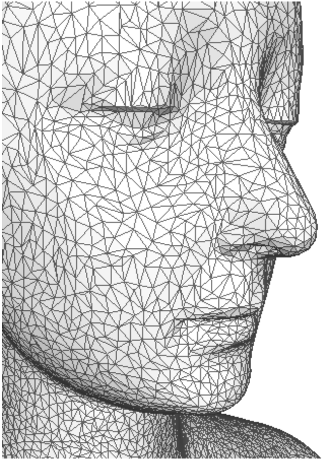


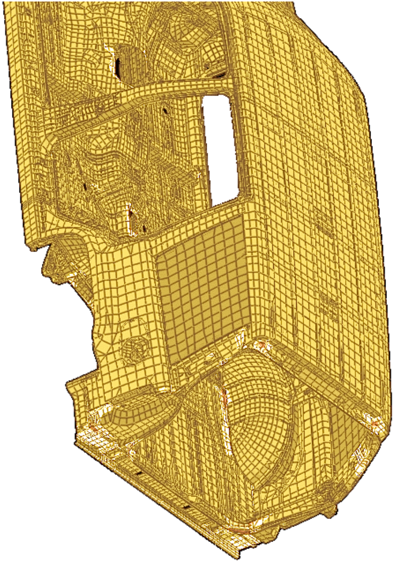


Meshing: Centroidal Voronoi Iteration

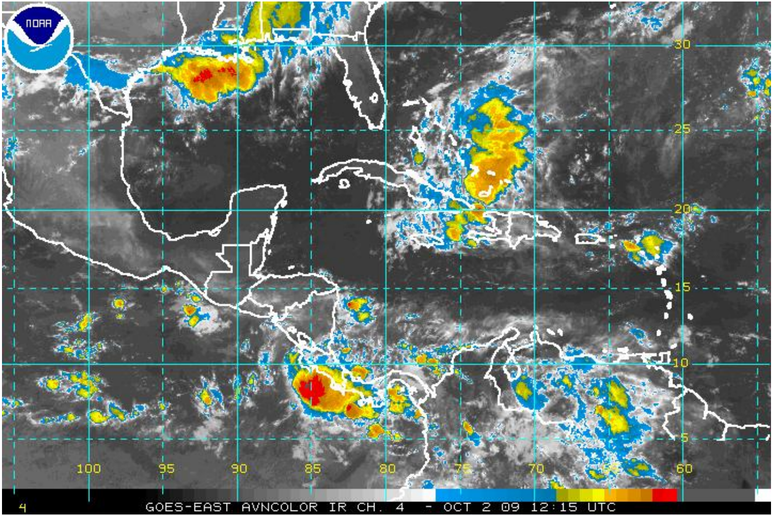


Meshing: Problems with Surfaces

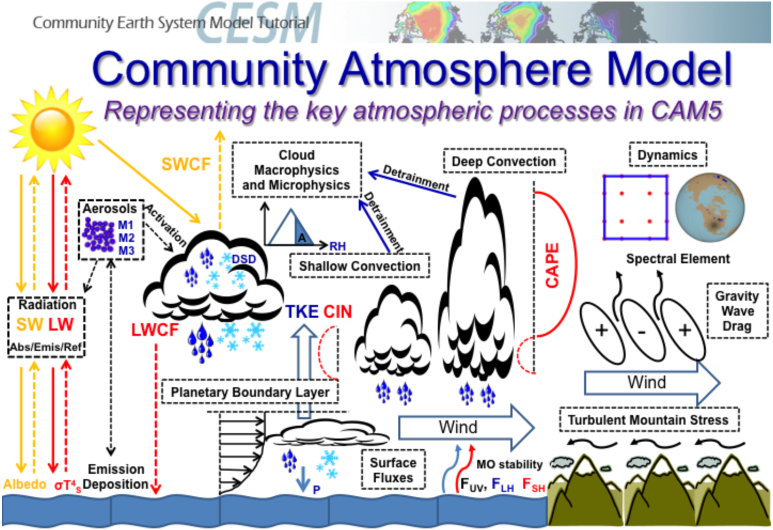




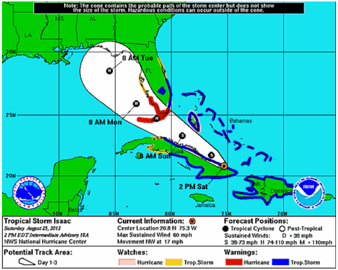
Models: Physics and Geometry of the Earth

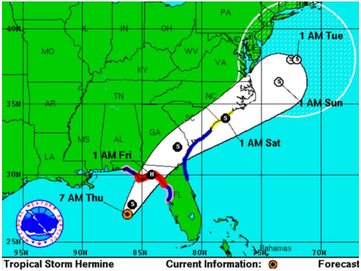


Models: Physical Processes to Model



Models: A Successful Prediction





Models: Millions of Nodes

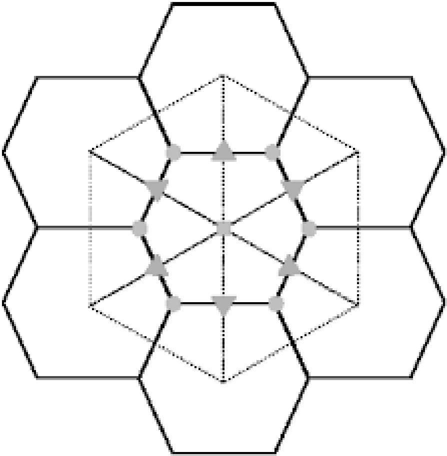
We are working with a climate modeling group at Los Alamos National Laboratory, whose **MPAS** software simulates the interactions of the atmosphere, ocean, and land over the entire globe.

They currently use meshes whose elements are about 15 kilometers on a side, or roughly 200 square kilometers in size. The surface area of the earth is about 510 million square kilometers; we need about 2 million elements, defined by nodes for which we can confidently say that they are about 15 kilometers apart.

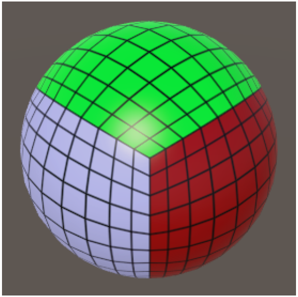
MPAS
Model for Prediction Across Scales

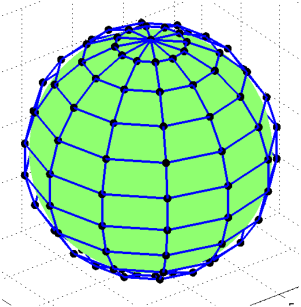
http://mpas-dev.github.io/

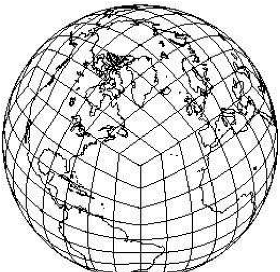
Models: Transport becomes Local Trading

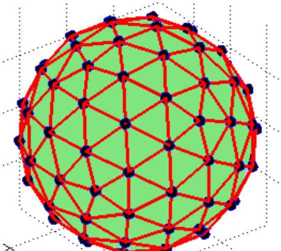


First Draft: Sphere Meshes



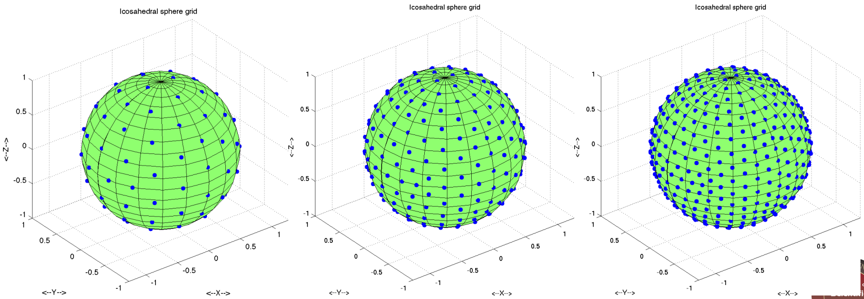






First Draft: Bisection of Icosahedral Grid

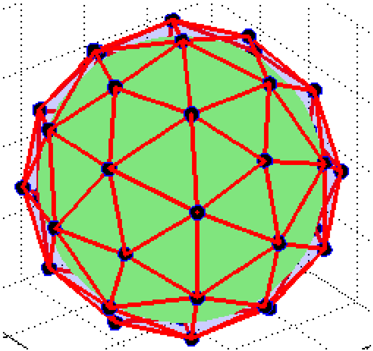
The 12 vertices of the icosahedron are perfectly separated on the sphere. If we triangulate these vertices, we get 20 faces. If we bisect each edge, we can replace each face with four smaller ones, which are no longer congruent, and no longer "perfectly" placed. As we repeatedly refine this grid by bisection, the mesh degrades, but is still very acceptable as a starting point.

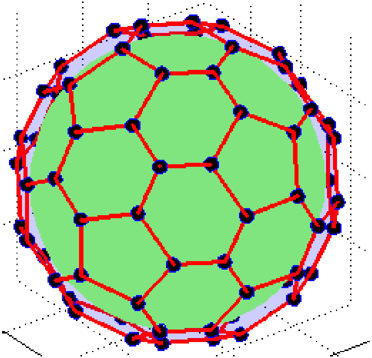


First Draft: STRIPACK-based Algorithm

```
Choose n initial points g using the bisection grid;

while ( true )
  v := Voronoi diagram ( g );
  Compute c(i) = centroid of Voronoi polygon for g(i);
  test = norm ( g - c );
  g <== c;
  if ( test <= tolerance ) break;

t = Delaunay triangulation ( g )

construct final mesh from g, v, t
```

First Draft: STRIPACK processes a 42 node grid



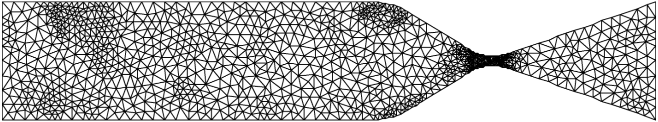


First Draft: Timing for One Iteration

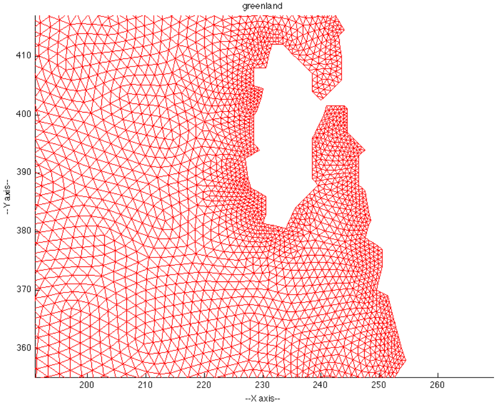
For a 15 kilometer element width on the Earth, using uniform elements, we need about 2,000,000 elements. Starting nodes are created by "bisecting" an icosahedral set of nodes. Times increasing like N^2.

BISECT	Nodes	Name	Time (seconds)
0	12		5.E-5
1	42		1.E-4
2	162		4.E-4
3	642		6.E-3
4	2,562		0.066
5	10,242		0.660
6	40,962	coarse	10.161
7	163,842	medium	170.798
8	655,362		3,207.510
9	2,621,442	fine	51,954.900
10	10,485,762		

TRIANGLE: Sequential Delaunay in Plane



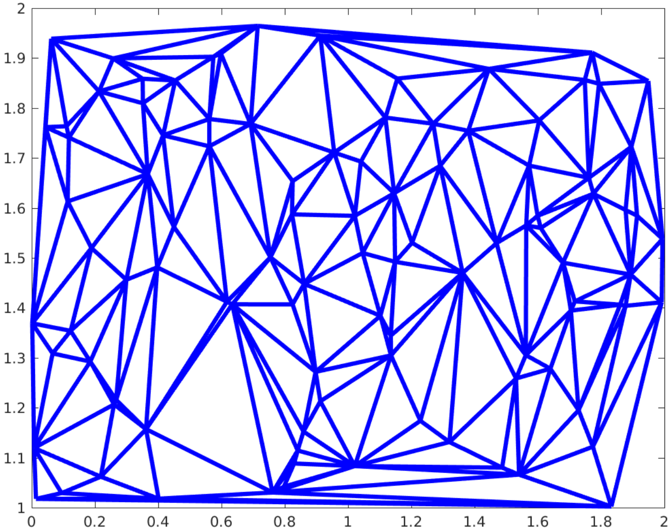
Source: Shewchuk (2005)



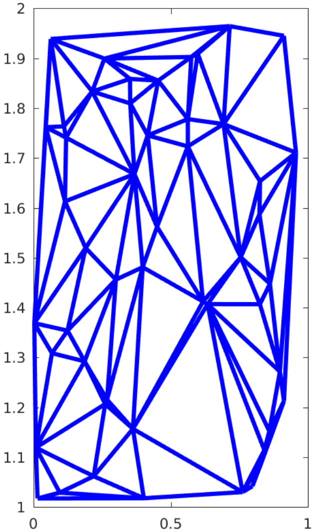
TRIANGLE: Same Problem Sizes as STRIPACK

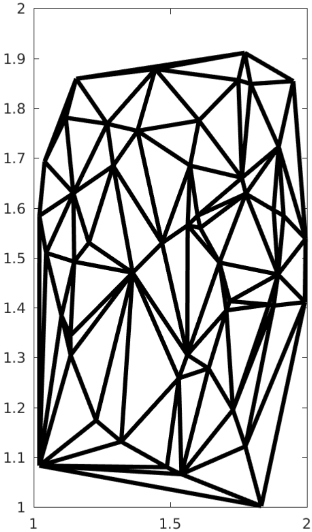
BISECT	Nodes	Name	STRIPACK Seconds	TRIANGLE Seconds
0	12		5.E-5	0.025
1	42		1.E-4	0.023
2	162		4.E-4	0.023
3	642		6.E-3	0.026
4	2,562		0.066	0.033
5	10,242		0.660	0.057
6	40,962	coarse	10.161	0.178
7	163,842	medium	170.798	0.707
8	655,362		3,207.510	2.649
9	2,621,442	fine	51,954.900	11.108
10	10,485,762		?	76.304

TRIANGLE: Opportunities for Parallelism?



TRIANGLE: Opportunities for Parallelism?





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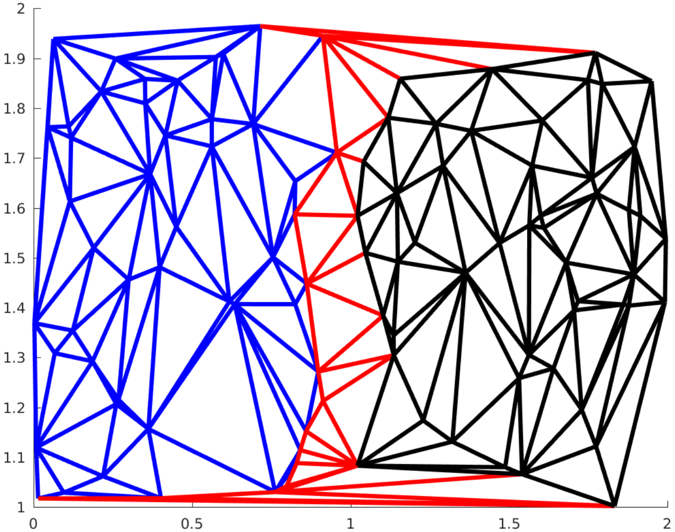
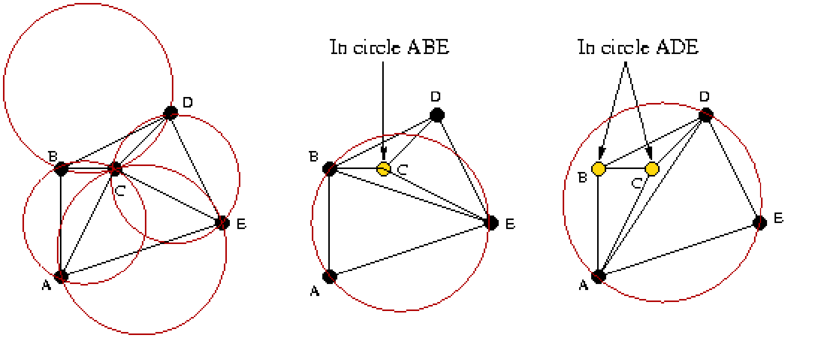
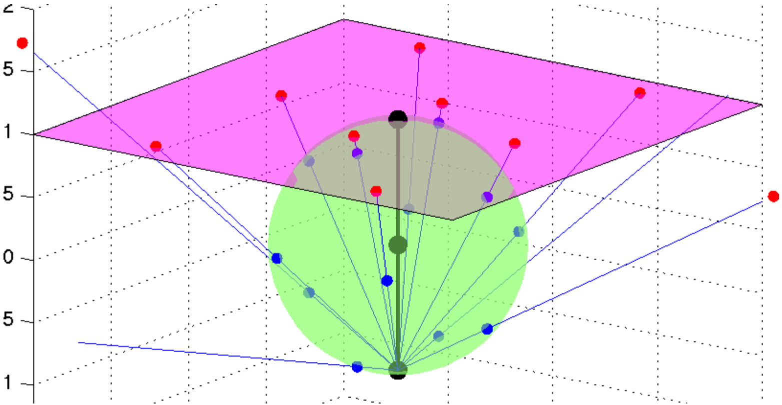


SPHERE: Empty Circumcircle Condition

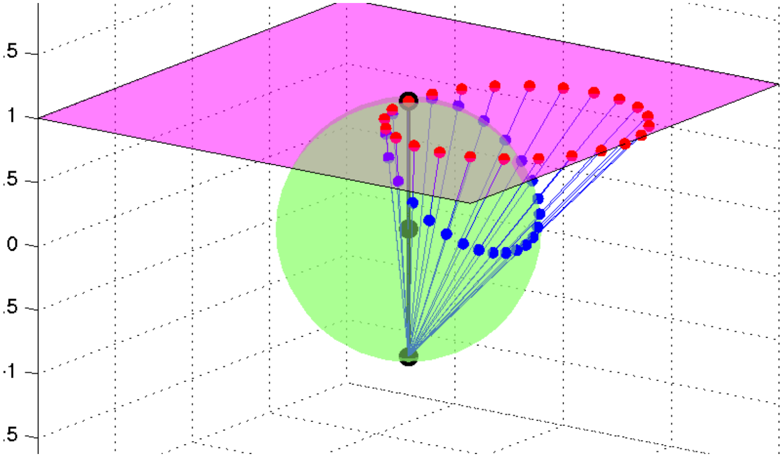


Delaunay Triangulation   Non-Delaunay Triangulation   Non-Delaunay Triangulation

SPHERE: Mapping between Plane and Sphere



SPHERE: Mapping Preserves Circles



SPHERE: Proposed CVT Algorithm

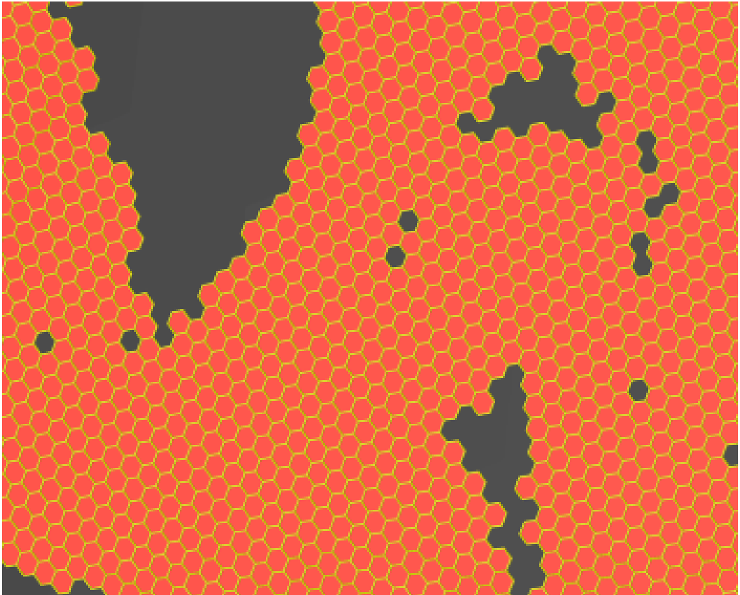
```
Choose n initial points g using the bisection grid;
Processor p* gets nodes g* + nodes g** of neighbors;

while ( tolerance < test )
  Stereograph g* + g** to plane;
  Compute local planar Delaunay triangulation ( g*+g** );
  Construct all spherical triangles that include any g* node;
  Accumulate c* = centroids of Voronoi polygons for g*;
  Compute local test = local norm ( g* - c* );
  Replace g* <== c*;
  Update node information with 6 neighbors;
  Gather local tests into global test;

Merge local Delaunay triangulations;
Compute Voronoi diagram;
Construct mesh (nodes, polygons, connections).
```

SPHERE: Speedups for local triangulation and merge

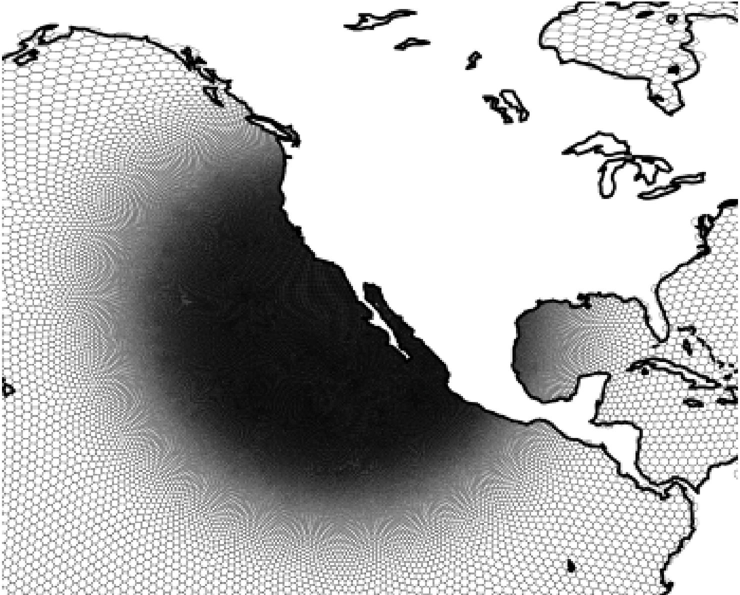
Computations for a “medium” grid of 163,842 nodes.

Algorithm	Procs	Regions	Speedup	Comment
STRIPACK	1	1	1	Used for local and merge.
MPI-SCVT L	1	2	57	Smallest code uses
MPI-SCVT L+M	1	2	21	2 processes.
MPI-SCVT L	42	42	4092	Called thousands of times.
MPI-SCVT L+M	42	42	37	Called once, at end.

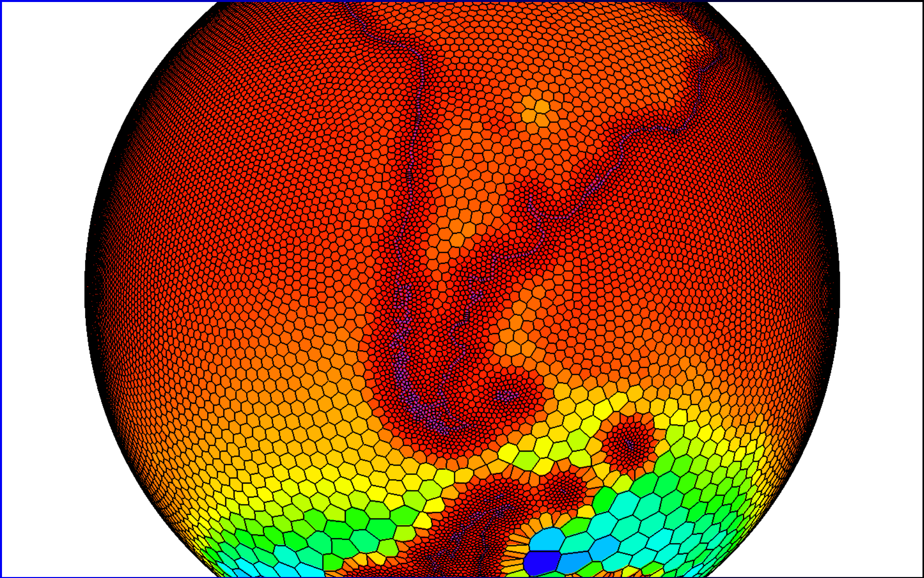
EXAMPLES: Uniform Mesh Near Florida Coast



EXAMPLES: Uniform Mesh Near California Coast



EXAMPLES: South America Land/Ocean Interface



MPI Issues

- The sphere surface naturally subdivides into 12, 42, 162, subregions;
- We can use any number of subregions (but at least 2!), but icosahedral bisection has advantages;
- For 2 million nodes, the 42 subregions leaves enough work for each MPI process;
- The regularity of the subregion connectivity means just 6 MPI Sends and Receives per process on each step;
- Only at the end of the iteration is a global MPI gather needed in order to assemble the mesh;
- If a nonuniform density is applied, the assignment of nodes to processors must be adjusted;

Credits

The work described here represents in part the PhD dissertation of **Doug Jacobsen**, while he was a student in the FSU Department of Scientific Computing.

Max Gunzburger and **Janet Peterson** were his advisors, leading a research group that included me.

The motivation for a smooth polygonal mesh of the earth came from **Todd Ringler** of Los Alamos National Laboratory.

Doug used to arrive at school even earlier than I did, and always had a question or mathematical issue or programming problem to discuss with me. Doug was in my introductory workshop on MPI; I showed him stereographic mapping, spherical geometry, the STRIPACK and TRIANGLE packages and how to use Delaunay information for Voronoi calculations.

The ideas for doing the Delaunay triangulation in parallel, for exploiting the icosahedral grid, and the computer implementation came entirely from him.

References

- Doug Jacobsen Max Gunzburger, Todd Ringler, John Burkardt, Janet Peterson, *Parallel algorithms for planar and spherical Delaunay construction with an application to centroidal Voronoi tessellations,* Geoscientific Model Development, Volume 6, 2013, 1353-1365.
- Qiang Du, Vance Faber, Max Gunzburger, *Centroidal Voronoi Tessellations: Applications and Algorithms,* SIAM Review, Volume 41, Number 4, December 1999, pages 637-676.
- Robert Renka, *Algorithm 772: STRIPACK: Delaunay Triangulation and Voronoi Diagram on the Surface of a Sphere,* ACM Transactions on Mathematical Software, Volume 23, Number 3, September 1997, pages 416-434.
- Todd Ringler, Lili Ju, Max Gunzburger, *A multiresolution method for climate system modeling: application of spherical centroidal Voronoi tessellations*, Ocean Dynamics, Volume 58, Number 5-6, 2008, pages 475-498.
- Jonathan Shewchuk, *Delaunay Refinement Algorithms for Triangular Mesh Generation,* Computational Geometry, Theory and Applications, Volume 23, May 2002, pages 21-74.

Conclusions

- Stereographic mapping allows us to transfer hard work on the sphere to simple work in the plane
- Mapping TRIANGLE results onto the sphere is faster than working directly on the sphere with STRIPACK;
- The planar Delaunay triangulation can be parallelized, including the merge step;
- Therefore, the sphere triangulation can be parallelized;
- This procedure provides an efficient parallel solution to a costly calculation;
- Nonuniform density? Constraints? Subregion meshing? (All can be handled)