PAlmetto **N**umber **T**heory **S**eries

Abstracts for December 5-6, 2009, Meeting

Plenary Speakers

Ben Brubaker (Massachusetts Institute of Technology).

Title: *The combinatorics of automorphic forms.*

Abstract: Spectral Eisenstein series play a crucial role in the study of automorphic forms. I'll discuss two combinatorial models for understanding the Fourier-Whittaker coefficients of Eisenstein series on higher rank groups (and their finite covers). Both models are related to finite dimensional highest weight representations of the associated Langlands dual group: the crystal graph and the 6-vertex model in statistical mechanics, also known as "square ice". I'll define these objects in the course of the talk and explain what they tell us about Eisenstein series and perhaps automorphic forms in general.

Mirela Ciperiani (University of Texas at Austin).

Title: *Tate-Shafarevich groups over anticyclotomic $\mathbb{Z}_p$ extensions.*

Abstract: Let E be an elliptic curve over $\mathbb{Q}$ with supersingular reduction at p and K an imaginary quadratic extension of $\mathbb{Q}$. We analyze the structure of the p-primary part of the Tate-Shafarevich group of E over the anticyclotomic $\mathbb{Z}_p$-extension K_∞/K by viewing it as a module over $\mathbb{Z}_p[\mathrm{Gal}(K_\infty/K)]$.

John Friedlander (University of Toronto).

Title: *Brinkmanship in the semi-linear sieve.*

Abstract: We discuss some recent applications of the semi-linear (otherwise known as half-dimensional) sieve. We provide a quick review of the necessary sieve background. The material is excerpted from the newly-finished book "Opera de Cribro" written jointly with Henryk Iwaniec.

Kannan Soundararajan (Stanford University).

Title: *Mean-values of multiplicative functions and applications.*

Abstract: The pioneering results of Wirsing and Halasz describe the situations when the mean-value of a multiplicative function can be large. Understanding the structure of such multiplicative functions has proved useful in applications to the Polya-Vinogradov inequality, weak subconvexity bounds for L-functions, and to the quantum unique ergodicity problem. I will try and explain some of these results and applications.

Invited Graduate Student Speaker

Vorrapan Chandee (Stanford University).

Title: *Bounding $|\zeta(1/2+it)|$ on the Riemann hypothesis.*

Abstract: In 1924 Littlewood showed that, assuming the Riemann Hypothesis, for large t there is a constant C such that $|\zeta(1/2+it)| \ll \exp(C\log t/\log\log t)$. Soundararajan and I show how the problem of bounding $|\zeta(1/2+it)|$ may be framed in terms of minorizing the function $\log((4+x^2)/x^2)$ by functions whose Fourier transforms are supported in a given interval, and drawing upon recent work of Carneiro and Vaaler we find the optimal such minorant. Thus we establish that any $C > (\log 2)/2$ is permissible in Littlewood's result. The talk will be based on this joint work with Soundararajan.

Contributed Talks

Dan Baczkowski (Washington and Lee).

Title: *Counting lattice points close to smooth curves*

Abstract: (joint work with O. Trifonov) We provide an introduction and history of some recent applications of this method. In general, a smorgasbord of problems have erupted from this topic. Our work provides a generalization to the previous literature for curves in two dimensions. The improvements given in this paper are followed with an application to a problem regarding F_q-rational points on curves of small genus over the finite field F_q.

Byungchul Cha (Muhlenberg College).

Title: *Growth rate of the summatory function of Möbius function in function fields.*

Abstract: It is known that a certain linear independence property of zeros of Riemann zeta function is related to the growth rate, as $x \to \infty$, of the summatory function $M(x) = \sum_{n\le x}\mu(n)$ of the Möbius function $\mu(n)$. In this talk, we review some known results on the growth rate of $M(x)$ and then describe some progress toward constructing its counterpart in the function field setting.

Josh Cooper (University of South Carolina).

Title: *Tree reconstruction and a Waring-type problem on partitions.*

Abstract: The "line graph" of a graph G is a new graph $L(G)$ whose vertices are the edges of G, with a new edge in $L(G)$ from e to f if e and f were incident in G. Graham's Tree Reconstruction Conjecture says that, if T is a tree (a connected, acyclic graph), then the sequence of sizes of the iterated line graphs of T uniquely determine T. That is, T can be reconstructed from $\{|L^{(j)}(G)|\}_{j=0}^{\infty}$, where $L^{(0)}(G) = G$ and $L^{(j+1)}(G) = L(L^{(j)}(G))$. Call two trees equivalent if they yield the same sequence; we call the resulting equivalence classes "Graham classes." Clearly, the conjecture is equivalent to the statement that the number of Graham classes of n-vertex trees is equal to the number of isomorphism classes of such trees, which is known to be about 2.955765^n.

We show that the number of Graham classes is at least superpolynomial in n (namely, $exp(c\log n^2)$) by converting the question into the following Waring-type problem on partitions. For a partition $\lambda = \{\lambda_1, \ldots, \lambda_k\}$ of the integer n and a degree d polynomial $f \in \mathbb{Z}[x]$, define $f(\lambda) = \sum_{j=1}^{k} f(\lambda_j)$. We show that the range of $f(\lambda)$ over all partitions λ of n grows as $\Omega(n^{d-1})$. The proof employs a well-known family of solutions to the Prouhet-Tarry-Escott problem. Strong evidence suggests the conjecture that the size of the range is actually $\Theta(n^d)$.

Johnson Jia (University of Michigan).

Title: *A p-integral Yoshida lift and non-vanishing mod p.*

Abstract: Yoshida lift is an instance of a theta lift in the theory of automorphic forms. In this case, we are sending a pair of automorphic forms on a definite quaternion algebra to a Siegel modular form of genus 2. I hope to give a quick review of the Yoshida lift and outline the strategy used in showing that (a refined version of) the Yoshida lift preserves integral structures and is in fact non-zero mod p.

Renling Jin (College of Charleston).

Title: *An answer to a question of Peter Hegarty and Mel Nathanson*

Abstract: A compact set K of the plane is said to satisfy the square property if for every point x there is a point y in K such that $x - y$ is an integer lattice point. For example, the unit square has this property. The question is that whether such K must contain two distinct points u and v such that $u - v$ is an integer lattice point, which is neither vertical nor horizontal. We will answer this question and explain the idea of the proof.

Neil Lyall (University of Georgia).

Title: *Simultaneous optimal return times*

Abstract: Let $P \in \mathbb{Z}[t]$ with $P(0) = 0$. It is a striking and elegant fact (proved independently by Furstenberg and Sárkőzy) that any subset of the integers of positive upper density necessarily contains two distinct elements whose difference is given by $P(t)$ for some $t \in \mathbb{Z}$. Using Fourier analysis we establish quantitative bounds for the following strengthening of this result:

> *Let $\varepsilon > 0$, then there exists N_ε such that if $N \geq N_\varepsilon$ and $A \subseteq \{1, \ldots, N\}$, then there exists $t \neq 0$ such that A contains at least $|A|^2/N - \varepsilon$ pairs of elements whose common differences are all equal to $P(t)$.*

We will also discuss the problem of finding simultaneous ε- optimal return times for a given collection of polynomials $P_1, \ldots, P_\ell \in \mathbb{Z}[t]$. Joint work with Ákos Magyar (University of British Columbia).

Ethan Smith (Michigan Technological University).

Title: *Average Frobenius distribution for elliptic curves defined over* number fields.

Abstract: Given an integer r and an elliptic curve E, the fixed trace Lang-Trotter Conjecture concerns the number of primes p up to x with trace of Frobenius $a_p(E)$ equal to r. In this talk, we will discuss a generalization of the conjecture to the setting of number fields and a result that says this conjecture is true "on average" when the number field is Galois over the rationals. This work builds on previous papers by Fouvry and Murty, David and Pappalardi, and Faulkner, James, King, and Penniston. This is joint work with Kevin James.

Frank Thorne (Stanford University).

Title: *Analytic properties of Shintani zeta functions.*

Abstract: The Shintani zeta function is a Dirichlet series which counts cubic rings, or equivalently, orbits of $\mathrm{SL}_2(\mathbb{Z})$ on the lattice of integral binary cubic forms. Shintani proved that the zeta function has an analytic continuation with a functional equation, without any Euler product or simple representation in terms of Euler products.

In this talk, we will examine the Shintani zeta function from an analytic point of view. In particular, we will ask questions about the distribution of its zeroes. The talk will concentrate on more open questions and possible approaches than proved results. Where applicable, we will also consider these questions in a more general setting.

Nathan Walters (University of Georgia).

Title: *Structure in sparse difference sets.*

Abstract: Many familiar theorems in various areas of mathematics have the following common feature: The set of differences from a sufficiently large set contains non-trivial structure. Specifically, we investigate the appearance of dilations of given sets. Using a slight simplification of the arguments of Croot, Ruzsa and Schoen we are able to establish the following result regarding a natural multidimensional generalization of the concept of an arithmetic progression: If $A \subset [1;N]^d$ with $|A|/N^d \geq CN^{-1/l}$ then there exists $r \neq 0$ such that $\{rv_1, ..., rv_l\} \subset A - A$. Time permitting we may also discuss some polynomial variants of these results.

John Webb (University of South Carolina).

Title: *Arithmetic of the* 13*-regular partition function modulo* 3.

Abstract: A k-regular partition is a partition where none of the parts are divisible by k. In this talk, we will present some new results regarding congruences relating values of the 13-regular partition function modulo 3. In particular, we identify an infinite family of non-nested arithmetic progressions modulo arbitrary powers of 3 such that all values are divisible by 3, confirming a conjecture from a 2008 seven-author paper. The proof relies on modular forms, so much of the talk will be an introduction to properties of modular forms and certain standard operators.

Hui Xue (Clemson University).

Title: *Fourier coefficients of Hilbert modular forms of half-integral weight.*

Abstract: In some special cases, we present and explicit formula that relates the central values of L-functions and the Fourier coefficients of Hilbert modular forms half-integral weight. The formula is obtained by geometric means.