PAlmetto **N**umber **T**heory **S**eries

SCHEDULE OF ACTIVITIES

(Coffee and talks are in Room 412 of LeConte.)

SATURDAY, DECEMBER 8

12:00 COFFEE

12:30 **Jeff Lagarias** (University of Michigan), *Integer orbits of discrete groups*

1:30 **Carrie Finch** (Washington and Lee), *On the irreducibility of a combination of polynomials which are the products of distinct cyclotomic polynomials*

1:55 **Mike Mossinghoff** (Davidson College), *Iterated maps and* Z*-numbers*

2:20 COFFEE BREAK

2:40 **Dan Goldston** (San Jose State University), *Small gaps between primes*

3:45 **Kostya Oskolkov** (University of South Carolina), *More on the differentiability of Riemann's 'non-differentiable' function*

4:30 COFFEE BREAK

4:50 **Greg Dresden** (Washington and Lee), *Binet-type formulas for* r*-generalized Fibonacci numbers*

5:15 **Ernie Croot** (Georgia Tech), *The structure of sets with few three-term arithmetic progressions*

6:00 DINNER

The organizers thank the National Science Foundation, the National Security Agency and the Mathematics Department at the University of South Carolina for their support.

SUNDAY, DECEMBER 9

8:00 COFFEE AND DOUGHNUTS

8:30 **Kathrin Bringmann** (University of Minnesota), *Hypergeometric series, automorphic forms, and mock theta functions*

9:30 **Ethan Smith** (Clemson University), *Finite field elements of high order arising from modular curves*

9:55 COFFEE BREAK

10:15 **Tom Tucker** (University of Rochester), *A dynamical Mordell-Lang theorem for split quadratic polynomial maps*

11:15 **Hiren Maharaj** (Clemson University), *A p-adic approach to distance graphs*

12:00 END OF CONFERENCE

ABSTRACTS

KATHRIN BRINGMANN, University of Minnesota, *Hypergeometric series, automorphic forms, and mock theta functions*

We will describe some of the connections between hypergeometric series and automorphic forms. The literature on examples of hypergeometric series that are related to modular forms is extensive, and the pursuit of further of these and their interpretation is an active area of research due to their applications to many areas of mathematics and to physics. However, the proofs of these scattered results fall far short of a comprehensive theory to describe the interplay between hypergeometric series and automorphic forms. The situation is further complicated by the mock theta functions, a collection of 22 q-series defined by Ramanujan in his last letter to Hardy. Though they resemble modular q-series, these functions do not arise as minor modifications of the Fourier expansions of modular forms. Recently, much light has been shed on the nature of Ramanujan's mock theta functions and it is now known that these functions are the holomorphic parts of weight $1/2$ weak Maass forms, and a clearer picture is beginning to emerge of which modular forms and Maass forms arise from basic hypergeometric series. We will describe part of those results.

Sunday, 8:30-9:30

ERNIE CROOT, Georgia Tech., *The structure of sets with few three-term arithmetic progressions*

We show that any subset of F_p having the minimal number of three-term arithmetic progressions for a given density must be a level set of certain sumsets, and then we use this to show that any such minimal S is therefore approximately a sumset.

Saturday, 5:15-6:00

GREG DRESDEN, (Washington and Lee University,) *Binet-type formulas for r-generalized Fibonacci numbers*

We are all familiar with the Binet formula for the Fibonacci numbers, $F_n = \frac{1}{\sqrt{5}}\alpha^n - \frac{1}{\sqrt{5}}\beta^n$, for $\alpha > \beta$ the roots of the polynomial $x^2 - x - 1$. It is interesting to note, by the way, that one really only needs the larger root, α, to generate the Fibonaccis, in that $F_n = \text{round}\left(\frac{1}{\sqrt{5}}\alpha^n\right)$ holds for all values of $n \geq 0$.

The r-generalized Fibonacci numbers (also called the Tribonaccis, Tetranaccis, etc.) satisfy the recurrence relation $G_n = G_{n-1} + G_{n-2} + \cdots + G_{n-r}$, and there are similar Binet-type formulas for these numbers, involving the sum of nth powers of the r roots of $x^r - x^{r-1} - x^{r-2} - \cdots - 1$. What is surprising is that (given appropriate initial values) these Binet-type formulas can be replaced with just the rounded value of the first term.

Saturday, 4:50-5:15

CARRIE FINCH, Washington & Lee University, *On the irreducibility of a combination of polynomials which are the products of distinct cyclotomic polynomials*

For each natural number j, define $\gamma_j(x) = x^j + x^{j-1} + \cdots + x + 1$. We investigate the irreducibility of combinations of polynomials of the form $\gamma_j(x)$, such as the sum $\gamma_{n_1}(x) + \gamma_{n_2}(x) + \cdots + \gamma_{n_k}(x)$ (with the n_i's distinct natural numbers) and the combination $\gamma_{n_1}(x) + x^m\gamma_{n_2}(x)$ (with m a natural number). This is joint work with Lenny Jones.

Saturday, 1:30-1:55

DAN GOLDSTON, San Jose State, *Small Gaps Between Primes*

I will talk about my recent work with Pintz and Yildirim which proves that there are pairs of primes very close together compared to the average distance between consecutive primes. Assuming a conjecture of Elliott and Halberstam that primes are evenly distributed among arithmetic progressions, our method proves that there are infinitely often pairs of primes differing by 16 or less. Twin primes are pairs of primes that are two apart, such as 3 and 5, or 29 and 31, and the twin prime conjecture is that there are infinitely many twin primes. Thus our method gives conditionally a result pretty close to the twin prime conjecture This work has had its share of media attention, and even generated a song on public television. I will show a few examples of this publicity and how mathematics is covered in the media.

Saturday, 2:40-3:45

JEFF LAGARIAS, University of Michigan, *Integer Orbits of Discrete Groups*

This talk discusses integer orbits of discrete groups in $GL(n, \mathbb{Z})$. It considers two examples: integer Apollonian circle packings and solutions of Pell's equation. Recent results of Bourgain, Gamburd and Sarnak show the orbits in the first case are "large" , in the sense that infinitely many elements of the orbit have a bounded number of prime factors. In the second case we formulate conjectures that the orbits are "small". Comparison of the conjectures with numerical data leads to some surprising speculations.

Saturday, 12:30-1:30

HIREN MAHARAJ, Clemson University, *A p-adic approach to distance graphs*

Given a set $D = \{d_1, d_2, \ldots\}$ of positive integers, one defines a *distance graph* with the set of integers $\mathbb{Z}$ as the vertex set and xy is an edge iff $|x - y| \in D$. In this talk we will describe joint work with Jeong-Hyun Kang in which we approach distance graphs by using p-adic methods. This allows us to give general bounds on the chromatic number that depend on the divisibility properties of the numbers d_i. Furthermore, the chromatic number is determined for large classes of distance graphs.

Sunday, 11:15-12:00

MICHAEL MOSSINGHOFF, Davidson College, *Iterated maps and Z-numbers*

Starting with a positive integer x, consider the iteration that sends x to $\lceil 4x/3 \rceil$ if $x \not\equiv 1 \bmod 3$ and halts if $x \equiv 1 \bmod 3$. For example, beginning with $x = 15$ we compute $15 \to 20 \to 27 \to 36 \to 48 \to 64 \to$ halt. Does this iteration halt on all initial inputs? This problem is related to the question of the existence of the so- called *Z-numbers* studied by Mahler and others. For the case of $4/3$, this problem asks if there exists a positive real number z such that the fractional part of $z \cdot (4/3)^n$ lies in $[0, 1/3)$ for every $n \geq 1$. We consider these questions for certain rational numbers p/q with $q < p < q^2$, and describe some algorithms developed to search for Z- numbers, or, alternatively, for initial values x for which a corresponding iteration never halts. For example, in the case where $p/q = 4/3$, we find that a Z- number, if it exists, must exceed 10^{32}. This is joint work with Artūrus Dubickas.

Saturday, 1:55-2:20

KONSTANTIN OSKOLKOV, University of South Carolina, *More on the differentiability of Riemann's 'non-differentiable function*

Saturday, 3:45-4:30

ETHAN SMITH, Clemson University, *Finite Field Elements of High Order Arising from Modular Curves*

In this talk, we use the recursive modular curve constructions of Elkies to generate high order elements in finite fields.

Sunday, 9:30-9:55

TOM TUCKER, University of Rochester, *A dynamical Mordell-Lang theorem for split quadratic polynomial maps*

We will prove the dynamical Mordell-Lang theorem for maps of the form $f : A^n \mapsto A^n$ where f acts as a quadratic polynomial in each coordinate. The theorem says that if a subvariety V of A^n intersects the orbit of a point P under the action of f in infinitely many points, then V must contain an f–periodic subvariety of positive dimension.

Sunday, 10:15-11:15
