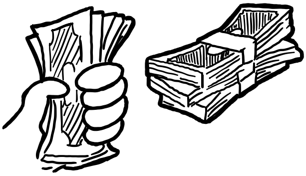
FINANCIAL APPLICATIONS (OF EXPONENTIAL FUNCTIONS)



OBJECTIVES:

USE OUR KNOWLEDGE OF EXPONENTIAL FUNCTIONS TO MODEL FINANCE-RELATED SCENARIOS & CHECK THE "RULE OF 70"

DEFN. AN AMOUNT OF MONEY P_0 IS DEPOSITED INTO AN ACCOUNT PAYING INTEREST AT A RATE OF r% / YEAR. LET P BE THE BALANCE OF THE ACCOUNT AFTER t YEARS.

- IF INTEREST IS COMPOUNDED ANUALLY THEN $P = P_0(1+r)^t$ ⟶ INTEREST IS APPLIED TO YOUR ACCOUNT YEARLY
- IF INTEREST IS COMPOUNDED CONTINUOUSLY THEN $P = P_0 e^{rt}$ ⟶ INTEREST IS APPLIED TO YOUR ACCOUNT CONTINUOUSLY

EXAMPLE BANK OF GOGO (BOGOGO FOR SHORT) ADVERTISES A SAVINGS ACCOUNT WITH AN INTEREST RATE OF 8% / YR. YOU CREATE A SAVINGS ACCOUNT WITH THE INTENT OF DEPOSITING \$5000, AND NOT TOUCHING THIS MONEY FOR 3 YRS. AS YOU'RE CREATING YOUR ACCOUNT, GOGO ASKS IF YOU WANT INTEREST COMPOUNDED ANNUALLY OR CONTINUOUSLY. WHAT SHOULD YOU CHOOSE TO MAKE THE MOST MONEY?

Bank of GoGo

ANNUAL COMPOUNDING: $P = P_0(1+r)^t$

- $P_0 = 5000$
- $r = 0.08 = 8\%$

$P = 5000(1.08)^t$

SO AFTER 3 YRS, BALANCE IS

$P = 5000(1.08)^3 = \boxed{\$6298.56}$

CONTINUOUS COMPOUNDING: $P = P_0 e^{rt}$

- $P_0 = 5000$
- $r = 0.08$ (AS BEFORE)

SO WE HAVE $P = 5000e^{.08t}$

AND AFTER 3 YRS, BALANCE IS:

$P = 5000e^{.08 \cdot 3} = \boxed{\$6356.25}$

SO YOU SHOULD PROBABLY CHOOSE TO HAVE YOUR INTEREST COMPOUNDED CONTINUOUSLY.

- WHAT IF I'M ONLY LETTING MONEY SIT & COLLECT INTEREST FOR 1 YR? DOES MY CHOICE CHANGE?
- ... WHAT ABOUT 6 MONTHS? ($t = 1/2$)

→ THINK OF.. DOUBLING TIME OF AN INVESTMENT

DEFN. THE DOUBLING TIME OF AN EXPONENTIALLY INCREASING QUANTITY IS THE TIME REQUIRED FOR THE QUANTITY TO DOUBLE.

THE HALF-LIFE OF AN EXPONENTIALLY DECAYING QUANTITY IS THE TIME REQUIRED FOR THE QUANTITY TO BE REDUCED BY A FACTOR OF ½.

↳ FOR HALF LIFE YOU CAN THINK OF RADIOACTIVE DECAY

THE RULE OF 70 IS A WAY TO ESTIMATE THE DOUBLING TIME OF AN INVESTMENT.

IT SAYS THAT GIVEN AN INTEREST RATE OF $n\%$ COMPOUNDED ANNUALLY, THE DOUBLING TIME OF AN INVESTMENT IS APPROXIMATELY $\frac{70}{n}$ YEARS.

... IS THIS TRUE?

EX. CALCULATE THE DOUBLING TIME D FOR AN INVESTMENT P_0 WITH INTEREST RATE:

- 2% COMPOUNDED ANNUALLY

WHAT WE WANT: VALUE OF t (IN YRS) FOR WHICH $P = 2P_0$.

WE'LL HAVE:

$P = P_0(1.02)^t$ BUT WE WANT t SO THAT $2P_0 = P_0(1.02)^t$ ← DIVIDE BOTH SIDES BY P_0! (WE KNOW OUR INITIAL INVESTMENT IS NONZERO)

Dividing by P_0 gives: $2 = (1.02)^t$ NOW SOLVE FOR t... To DO SO YOU CAN LOGARITHMEATE

$$\ln(2) = \ln(1.02^t)$$
$$= t \cdot \ln(1.02) \quad \text{so} \quad t = \frac{\ln(2)}{\ln(1.02)} \approx \text{APPROX } D = \boxed{35.003 \text{ YRS.}}$$

- 3% COMPOUNDED ANNUALLY

NOW WE HAVE $P = P_0(1.03)^t$, AND WE WANT t SO THAT $P = 2P_0$

$2P_0 = P_0(1.03)^t$ ⟶ DIVIDE BOTH SIDES BY P_0, THEN LOGARITHMEATE

$2 = 1.03^t$ ⟶ $\ln(2) = \ln(1.03^t)$ so $\ln(2) = t\ln(1.03)$

$$t = \frac{\ln(2)}{\ln(1.03)} \approx D = \boxed{23.5 \text{ YRS}}$$

- 4% COMPOUNDED ANNUALLY

- 5% COMPOUNDED ANNUALLY