

MORE EQUATION SOLVING

P1



SOLVE FOR X IN THE FOLLOWING:

$$\log_4(x^2 - 2x) = \log_4(5x - 12)$$

IN THIS CASE... THE UNKNOWN IS TRAPPED INSIDE OF \log_4
TO GET RID OF THIS, WE CAN 'EXPONENTIATE' WITH 4 AS OUR BASE.

$$4^{\log_4(x^2 - 2x)} = 4^{\log_4(5x - 12)}$$

SO WE'RE LEFT WITH $x^2 - 2x = 5x - 12 \rightarrow x^2 - 7x + 12 = 0$

YOU CAN FACTOR OR USE QUAD FORMULA.

$$x^2 - 7x + 12 = (x - 4)(x - 3) = 0$$

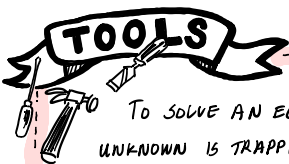
$$\Rightarrow x = 4 \text{ OR } x = 3$$

* AT THIS POINT ITS A GOOD IDEA TO CHECK THESE SOLUTIONS TO MAKE SURE I'M NOT TAKING LOGARITHM OF A NEGATIVE NUMBER!

IT TURNS OUT THAT BOTH

$$x = 3 \text{ \& } x = 4 \text{ WORK.}$$

SOLUTIONS: $\boxed{x = 3 \text{ \& } x = 4}$



TO SOLVE AN EQUATION WHERE THE UNKNOWN IS TRAPPED INSIDE OF \log_a , WE CAN 'EXPONENTIATE' WITH BASE a TO MOVE THE UNKNOWN OUT OF \log_a ...

$$\text{ie. } \log_a(x) = 1$$

$$\Rightarrow a^{\log_a(x)} = a^1 \text{ so } x = a^1$$



SOLVE FOR X:

$$\log(6x) - \log(4 - x) = \log(3)$$

RECALL: LOG WITHOUT A BASE IS ASSUMED TO BE \log_{10} .

THIS PREVENTS ME FROM EXPONENTIATING IMMEDIATELY.

NEED TO COMBINE LEFT SIDE INTO A SINGLE LOGARITHM. USING LOG RULES, WE KNOW WE CAN WRITE $\log(6x) - \log(4 - x)$ AS $\log\left(\frac{6x}{4 - x}\right)$

NOW I CAN EXPONENTIATE USING 10 AS MY BASE.

$$10^{\log\left(\frac{6x}{4 - x}\right)} = 10^{\log(3)}$$

SIMPLIFY $\frac{6x}{4 - x} = 3$

NOW SOLVING FOR X SHOULDNT BE TOO BAD.

$$6x = 3(4 - x)$$

$$6x = 12 - 3x \Rightarrow \boxed{x = \frac{4}{3}}$$



Assume $a > 0$. Solve for t .

$$\log_a(t^2 - 1) = \log_a(5t + 7)$$

WE CAN START BY EXPONENTIATING WITH BASE a .

$$a^{\log_a(t^2 - 1)} = a^{\log_a(5t + 7)}$$

WHICH SIMPLIFIES TO $t^2 - 1 = 5t + 7$

$$t^2 - 5t - 8 = 0 \leadsto \text{QUADRATIC FORMULA OR CALCULATOR}$$

CALCULATOR GIVES THE FOLLOWING:

$$t = -1.2749 \quad \& \quad t = 6.2745$$

YOU SHOULD CHECK THESE ANSWERS TO MAKE SURE THEY WORK WHEN WE PLUG BACK INTO EQN.

SOLUTIONS:

$$t = 6.2745$$

$$\& \quad t = -1.2749$$



NOTE: IF THERE'S A \log_a IN YOUR EQN, P^2
YOU SHOULD EXPONENTIATE WITH BASE a .



SOLVE FOR x :

$$B = Pe^{rx}$$

NOTE FIRST: IN ORDER TO SOLVE FOR x , WE DO NOT NEED TO KNOW WHAT $B, P, \text{ or } r$ ARE.

THEY'RE JUST NUMBERS AND WE'LL TREAT THEM AS SUCH.

WE CAN LOGARITHMEATE:

$$\ln(B) = \ln(Pe^{rx})$$

$$\text{so } \ln(B) = \ln(P) + rx \quad \dots \text{ISOLATE } x$$

$$\ln(B) - \ln(P) = rx$$

SO

$$\frac{\ln(B) - \ln(P)}{r} = x$$

* BE VERY CAREFUL HERE.

$$\ln(Pe^{rx}) \neq rx \ln(Pe)$$

$$\ln(Pe^{rx}) = \ln(P) + \ln(e^{rx})$$

$$= \ln(P) + rx$$

LAST CLASS WE SOLVED $12 = 5e^{5x}$ BY DIVIDING BOTH SIDES BY 5, THEN LOGARITHMEATING. WE CAN DO THE SAME HERE.

SOLVE FOR x :

$$\log_2(x) = 1 - \log_2(x-3)$$

REWRITING WILL MAKE THINGS EASIER:

$$\log_2(x) + \log_2(x-3) = 1$$

USE LOG RULES TO
COMBINE

$$\log_2(x(x-3)) = 1 \rightarrow \log_2(x^2 - 3x) = 1$$

NOW EXPONENTIATE WITH BASE 2.

$$2^{\log_2(x^2 - 3x)} = 2^1$$

WHICH GIVES US:

$$x^2 - 3x = 2 \rightarrow x^2 - 3x - 2 = 0$$

USE CALCULATOR OR QUADRATIC FORMULA.

$$x = -.561 \text{ OR } x = 3.561 \rightarrow \text{CHECK THESE ANSWERS!}$$

NOTICE THAT IF $x = -.561$, WE HAVE $\log_2(-.561)$, WHICH DOESN'T EXIST.

$$\text{SO } \boxed{x = 3.561}$$

IS THE ONLY SOLUTION.

LAST CLASS WE SAW THE FOLLOWING FACT:

$$\text{IF } a^x = a^y, \text{ THEN } x = y.$$

WITH THIS IN MIND, WE HAVE:

$$\text{FACT: IF } \log_b(x) = \log_b(y) \text{ THEN } x = y.$$

SOLVE FOR y :

$$\log_7(y^2 - 3) = \log_7(2y)$$

USING THE FACT ABOVE, WE KNOW WE MUST HAVE $y^2 - 3 = 2y$.

$$\rightarrow y^2 - 2y - 3 = 0, \text{ SOLVE THIS QUADRATIC}$$

$$\dots \text{ IT FACTORS AS } (y+1)(y-3) = 0.$$

SO WE HAVE $y = -1$ OR $y = 3$. CHECKING OUR ANSWERS, WE SEE $y = -1$ WON'T WORK,BECAUSE ON THE RIGHT SIDE OF EQN WE'LL GET $\log_7(-2)$.

$$\text{SO OUR ONLY SOLUTION IS } \boxed{y = 3}$$