MORE EQUATION SOLVING

CONTINUED FROM LAST CLASS...

Ex SOLVE FOR x IN THE FOLLOWING..

$$\log_4(x^2-2x) = \log_4(5x-12)$$

HOW IS THIS DIFFERENT FROM WHAT WE WERE DOING ON MONDAY?

IN THIS CASE... THE UNKNOWN IS TRAPPED INSIDE OF $\log_4$

TO GET RID OF THIS, WE CAN "EXPONENTIATE" WITH 4 AS OUR BASE.

$$4^{\log_4(x^2-2x)} = 4^{\log_4(5x-12)}$$

SO WE'RE LEFT WITH $x^2-2x = 5x-12 \longrightarrow x^2-7x+12=0$

YOU CAN FACTOR OR USE QUAD. FORMULA.

$$x^2-7x+12 = (x-4)(x-3) = 0$$

$$\Rightarrow x=4 \text{ OR } x=3$$

* AT THIS POINT IT'S A GOOD IDEA TO CHECK THESE SOLUTIONS TO MAKE SURE I'M NOT TAKI LOGARITHM OF A NEGATIVE NUMBER!

IT TURNS OUT THAT BOTH $x=3$ & $x=4$ WORK.

SOLUTIONS: $\boxed{x=3 \text{ \& } x=4}$

TOOLS

TO SOLVE AN EQUATION WHERE THE UNKNOWN IS TRAPPED INSIDE OF $\log_a$, WE CAN "EXPONENTIATE" WITH BASE a TO MOVE THE UNKNOWN OUT OF $\log_a$..

ie. $\log_a(x) = 1$

$$\Rightarrow a^{\log_a(x)} = a^1 \quad \text{SO} \quad x=a^1$$

Ex SOLVE FOR x:

$$\underbrace{\log(6x)-\log(4-x)}_{} = \log(3)$$

RECALL: LOG WITHOUT A BASE IS ASSUMED TO BE $\log_{10}$.

THIS PREVENTS ME FROM EXPONENIATING IMMEDIATELY.

NEED TO COMBINE LEFT SIDE INTO A SINGLE LOGARITHM. USING LOG RULES, WE KNOW WE CAN WRITE $\log(6x)-\log(4-x)$ AS $\log\left(\frac{6x}{4-x}\right)$

NOW I CAN EXPONENTIATE USING 10 AS MY BASE.

$$10^{\log\left(\frac{6x}{4-x}\right)} = 10^{\log(3)} \xrightarrow{\text{SIMPLIFY}} \frac{6x}{4-x} = 3$$

NOW SOLVING FOR x SHOULDN'T BE TOO BAD..

$$6x = 3(4-x)$$

$$6x = 12-3x \Rightarrow \boxed{x = 4/3}$$

Ex Assume $a > 0$. Solve for t:

$$\log_a(t^2-1) = \log_a(5t+7)$$

NOTE: If there's a $\log_a$ in your eqn, you should exponentiate with base a.

We can start by exponentiating with base a.

$$a^{\log_a(t^2-1)} = a^{\log_a(5t+7)}$$

which simplifies to $t^2-1 = 5t+7$

$t^2 - 5t - 8 = 0$ ⇝ Quadratic formula or calculator

Calculator gives the following:

$t = -1.2749$ & $t = 6.2745$.

→ You should check these answers to make sure they work when we plug back into eqn.

SOLUTIONS:
$t = 6.2745$
& $t = -1.2749$

Ex Solve for x:

$$B = Pe^{rx}$$

NOTE FIRST: In order to solve for x, we do not need to know what B, P, or r are. They're just numbers, and we'll treat them as such.

We can logarithmeate:

$$\ln(B) = \ln(Pe^{rx}) \,*$$

so $\ln(B) = \ln(P) + rx$ ... isolate x

$$\ln(B) - \ln(P) = rx$$

so

$$\boxed{\frac{\ln(B) - \ln(P)}{r} = x}$$

* Be very careful here.
$\ln(Pe^{rx}) \neq rx\ln(Pe)$
$\ln(Pe^{rx}) = \ln(P) + \ln(e^{rx})$
$= \ln(P) + rx$

Last class we solved $12 = 5e^{3x}$ by dividing both sides by 5, then logarithmeating. We can do the same here.

Ex SOLVE FOR x:

$$\log_2(x) = 1 - \log_2(x-3)$$

REWRITING WILL MAKE THINGS EASIER:

$$\underbrace{\log_2(x) + \log_2}_{\text{USE LOG RULES TO COMBINE}}(x-3) = 1$$

$$\log_2(x(x-3)) = 1 \longrightarrow \log_2(x^2-3x) = 1$$

NOW EXPONENTIATE WITH BASE 2.

$$2^{\log_2(x^2-3x)} = 2^1$$

WHICH GIVES US:

$$x^2-3x = 2 \longrightarrow x^2-3x-2=0$$

USE CALCULATOR OR QUADRATIC FORMULA.

$x = -.561$ or $x = 3.561$ $\longrightarrow$ CHECK THESE ANSWERS!

NOTICE THAT IF $x=-.561$, WE HAVE $\log_2(-.561)$, WHICH DOESNT EXIST.

SO $\boxed{x=3.561}$ IS THE ONLY SOLUTION.

LAST CLASS WE SAW THE FOLLOWING FACT:

IF $a^x = a^y$, THEN $x=y$.

WITH THIS IN MIND, WE HAVE:

FACT: IF $\log_b(x) = \log_b(y)$ THEN $x=y$.

Ex SOLVE FOR y:

$$\log_7(y^2-3) = \log_7(2y)$$

USING THE FACT ABOVE, WE KNOW WE MUST HAVE $y^2-3 = 2y$.

$\longrightarrow y^2-2y-3=0$, SOLVE THIS QUADRATIC

... IT FACTORS AS $(y+1)(y-3)=0$.

SO WE HAVE $y=-1$ OR $y=3$. CHECKING OUR ANSWERS, WE SEE $y=-1$ WONT WORK, BECAUSE ON THE RIGHT SIDE OF EQN WE'LL GET $\log_7(-2)$.

SO OUR ONLY SOLUTION IS $\boxed{y=3}$