THE NATURAL LOGARITHM

SECTION 1.6

OBJECTIVES:

USE LOGARITHMS TO FIND SOLUTIONS TO EXPONENTIAL EQUATIONS

DEFN THE **NATURAL LOGARITHM** OF x, WRITTEN $\ln(x)$ IS THE POWER OF e NEEDED TO GET TO x.

ie: $\ln(x) = y \iff e^y = x$

FURTHER.. IN GENERAL, FOR ANY $a > 0$, WE HAVE:

$$\log_a(x) = y \iff a^y = x$$

↑ "log base a of x"

* see fact @ end of notes

NOTE: MANY ISSUES YOU MAY HAVE HAD WITH LOGARITHMS IN THE PAST CAN PROBABLY BE FIXED BY REMEMBERING THAT A LOGARITHM IS AN EXPONENT AND

$$\ln(x) = y \iff e^y = x$$

RECALL SOME PROPERTIES OF LOGARITHMS.

- $\log_x(AB) = \log_x(A) + \log_x(B)$
- $\log_x\left(\frac{A}{B}\right) = \log_x(A) - \log_x(B)$
- $\log_a(x^p) = p \cdot \log_a(x)$
- $\log_a(a^x) = x$
- $a^{\log_a(x)} = x$

→ ALSO NOTE: $\ln(e^x) = x$ & $e^{\ln(x)} = x$

$\ln(x)$ is not defined if x is negative or zero!

→ TO REMEMBER THIS: IS THERE ANY y SO THAT $5^y = 0$?

Ex LOGARITHMS CAN BE USED TO SOLVE EQNS WHEN THE UNKNOWN IS IN THE EXPONENT.

FIND x SUCH THAT $3^x = 10$

WE CAN "LOGARITHMEATE" (TAKE $\ln$ OF BOTH SIDES) TO GET x OUT OF THE EXPONENT.

$$\ln(3^x) = \ln(10)$$

$$\dots \quad x\ln(3) = \ln(10)$$

$$\dots \quad x = \frac{\ln(10)}{\ln(3)} \approx \boxed{2.0959}$$

→ NOTICE THAT $3^{2.0959\dots} \approx 10$.

Ex FIND x SUCH THAT $12 = 5e^{3x}$

→ ISOLATE EXPONENT...

(To do so we can divide both sides by 5)

$\frac{12}{5} = e^{3x}$ ⟶ $2.4 = e^{3x}$

→ : LOGARITHMEATE :

$\ln(2.4) = \ln(e^{3x})$

so $\ln(2.4) = 3x \cdot \ln(e)$ ⟶ WE KNOW $\ln(e) = 1$..

so $\ln(2.4) = 3x$

AND $x = \frac{\ln(2.4)}{3} \approx \boxed{0.2918}$

TOOLS

TO SOLVE AN EQUATION WHERE THE UNKNOWN IS IN THE EXPONENT, WE CAN "LOGARITHMEATE" (TAKE LOG OF BOTH SIDES) TO MOVE UNKNOWN OUT OF EXPONENT.

ie. $a^{5x+1} = 1$

$\Rightarrow \log_a(a^{5x+1}) = \log_a(1)$

$\Rightarrow 5x+1 = 0$

FACT: IF $a^x = a^y$, THEN $x = y$. (SAME BASES)

Ex SOLVE FOR x:

$3^x = 9^{x+5}$

$3^x = (3^2)^{x+5}$ so $3^x = 3^{2(x+5)}$

$3^x = 3^{2x+10}$ NOTICE: SAME BASES

$x = 2x + 10$

$-2x \quad -2x$

$-x = 10$ ⟶ so $\boxed{x = -10}$

Ex $4^{5-9x} = \frac{1}{8^{x-2}}$

ISSUE: I CANT WRITE 4 AS A POWER OF 8, & I CANT WRITE 8 AS A POWER OF 4.

... BUT I CAN WRITE THEM BOTH AS POWERS OF 2!

$(2^2)^{5-9x} = \frac{1}{(2^3)^{x-2}}$

$2^{10-18x} = \frac{1}{2^{3x-6}}$ ⟶ $2^{10-18x} = 2^{-3x+6}$

so $\boxed{x = \frac{4}{15}}$

Ex Solve for y:

$$2^{4y+1} - 3^y = 0$$

NOTE: CAN'T LOGARITHMEATE RIGHT AWAY. (CAN'T TAKE $\ln(0)$..)

SO..

$2^{4y+1} = 3^y$ → NOW LOGARITHMEATE

$\ln(2^{4y+1}) = \ln(3^y)$ → $(4y+1)\ln(2) = y\ln(3)$

NOW SOLVE FOR y...

$4y\ln(2) + \ln(2) = y\ln(3)$ → MOVE y TO ONE SIDE

$\ln(2) = y\ln(3) - 4y\ln(2)$ → FACTOR OUT A y

then..

$\ln(2) = y(\ln(3) - 4\ln(2))$

↑ divide both sides by this..

$$y = \frac{\ln(2)}{\ln(3) - 4\ln(2)} \approx -.41407\ldots$$

CAN WE COME UP WITH AN ALGORITHM FOR SOLVING EXPONENTIAL EQNS LIKE THESE?

FACT. (FROM COLLEGE ALGEBRA)

$\ln(x)$ AND e^x ARE INVERSES.

THIS MEANS

$\ln(e^x) = x$ AND $e^{\ln(x)} = x$.

INFACT, IN GENERAL..

$\log_a(x)$ & a^x ARE INVERSES.