Exponential Functions

Section 1.5

Objectives:

Apply knowledge of exp. functions to solve real-world problems

DEFN We say that P is an **exponential function of t** with **base a** if:

$$P = P_0 a^t$$

- P_0 = initial quantity (y-intercept) when $t=0$.
- a = factor by which P changes when t is increased by 1. We call this the **growth factor**

The growth factor a is given by:

$$a = 1 + r$$

where r is the (decimal) percent rate of change.

- If $a > 1$, we have **exponential growth**
- If $0 < a < 1$, we have **exponential decay**

... what if a=1?

KEY

A linear function has constant rate of change. An exponential function has constant percent, or relative, rate of change.

EX Suppose the body has an initial amount of 10mg adrenaline. Find formula(s) for A, the amount in mg at a time t minutes later if:

Linear

- A is increasing by 0.3mg/minute

In this case, A is a linear function of t, with initial value 10mg & slope 0.3mg.

$$A = 0.3t + 10$$

- A is decreasing by 0.3mg/min

Again, A is a linear function of t, initial value is 10mg & slope is -0.3mg

$$A = -0.3t + 10$$

Exp.

- A is increasing by 3%/minute

A is an exponential function of t. Initial amount is 10mg, so $P_0 = 10$mg.

$r = 0.03$, so $a = 1 + 0.03$.

$$A = 10 \cdot (1.03)^t$$

(exponential growth)

- A is decreasing by 3%/minute

Again, A is an exp. function of t with initial amount 10mg. $r = -0.03$, because A is decreasing by 3%/minute. So. $a = 1 + (-0.03) = 0.97$.

$$A = 10(0.97)^t$$

(exponential decay)

AT THIS POINT, IT'S HELPFUL TO RECALL SOME RULES OF EXPONENTS..

RULES

LET x, y BE SOME NONZERO REAL NUMBERS.

- $x^1 = x$
- $x^0 = 1$
- $x^{-n} = \frac{1}{x^n}$
- $(x^m)(x^n) = x^{m+n}$
- $\frac{x^m}{x^n} = x^{m-n}$
- $(x^m)^n = x^{mn}$
- $(xy)^n = x^n y^n$
- $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$

CAN YOU COME UP WITH EXAMPLES OF EACH OF THESE RULES IN ACTION?

NOTE. IN PRACTICE, THE MOST COMMONLY USED BASE IS e ($\approx 2.71828\ldots$), WHICH IS CALLED "THE NATURAL BASE." IT'S AN IRRATIONAL NUMBER SORT OF LIKE π, AND WILL PROBABLY BECOME YOUR FAVORITE NUMBER AFTER THIS CLASS! → will talk about this next class..

Ex FOR THE ANTIBIOTIC AMPICILLIN, APPROX. 40% OF THE DRUG IS ELIMINATED FROM THE BODY EVERY HOUR. A TYPICAL DOSE IS 250 mg.

LET $Q = f(t)$, WHERE Q = QUANTITY (IN mg) IN BLOODSTREAM AT t HOURS SINCE DRUG WAS GIVEN.

AT $t = 0 \ldots$ $Q = 250$ mg.

THE QUANTITY REMAINING AT THE END OF EACH HOUR IS 60% OF THE QUANTITY REMAINING THE HOUR BEFORE.

$$Q = f(0) = 250$$

$$Q = f(1) = 250 \cdot (.60) = 150 \text{ mg}$$

$$Q = f(2) = 150 \cdot (.60) = 90 \text{ mg} = 250(.6)^2$$

$$\vdots$$

SO AFTER t HOURS..

$$\boxed{Q = f(t) = 250(.60)^t}$$