RATES OF CHANGE – SECTION 1.3

OBJECTIVES:

- RECOGNIZE THE SIMILARITIES BETWEEN SLOPE & RoC
- BE ABLE TO COMPUTE AVG. RATE OF CHANGE ON INTERVAL FOR NONLINEAR FUNCTIONS

WE ALREADY KNOW A LITTLE ABOUT RATES OF CHANGE... FROM LAST CLASS RECALL: $y = mx + b$ (EQN OF A LINE)

m = SLOPE = RATE OF CHANGE

WHAT IF I HAVE DATA THAT I WANT TO MODEL, BUT THIS DATA DOESN'T FOLLOW A LINEAR TREND? IF I NEED TO KNOW THE SLOPE.. HOW COULD I FIGURE IT OUT?

"RATE OF CHANGE" = FANCY WORD FOR SLOPE

AS IT STANDS, WE (AS A CLASS) DO NOT KNOW HOW TO COMPUTE THE SLOPE OF A CURVE IN GENERAL. (WE'LL GET TO THIS EVENTUALLY!) BUT... WE CAN AT LEAST TRY TO APPROXIMATE IT!

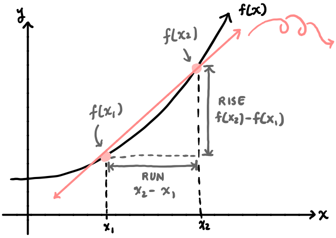
TO DO SO, WE'LL USE "AVERAGE RATE OF CHANGE"

DEFN. LET f BE A FUNCTION. FOR ANY TWO POINTS $(x_1, f(x_1))$ & $(x_2, f(x_2))$ THE **AVERAGE RATE OF CHANGE** OF f FROM x_1 TO x_2 IS GIVEN BY:

$$\text{AVG RATE OF CHANGE} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} \quad \text{OR, IF YOU PREFER:} \quad \frac{y_2 - y_1}{x_2 - x_1}$$

JUST LIKE HOW WE'D FIND THE SLOPE OF A LINE!

GOAL: APPROXIMATE SLOPE OF $f(x)$ FROM x_1 TO x_2 BY DRAWING A LINE THROUGH $f(x_1)$ & $f(x_2)$ AND FINDING THE SLOPE OF THE LINE.



I KNOW HOW TO FIND THE SLOPE OF THIS LINE!

THIS LINE HAS A NAME..

DEFN. A **SECANT LINE** IS A STRAIGHT LINE JOINING TWO POINTS ON A FUNCTION $f(x)$. THE SLOPE OF THE SECANT LINE CONNECTING $(x_1, f(x_1))$ & $(x_2, f(x_2))$ IS GIVEN BY

$$m = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

... WHICH WE KNOW IS THE AVERAGE RATE OF CHANGE

SLOPE OF SECANT LINE = AVERAGE RATE OF CHANGE BETWEEN TWO POINTS

EX A TENNIS BALL IS THROWN UP INTO THE AIR. YOU COLLECT THE FOLLOWING DATA:

t (sec)	0	1	2	3	4	5	6
y (ft)	6	90	142	162	150	106	30

HEIGHT y OF THE BALL ABOVE THE GROUND t SECONDS AFTER IT WAS THROWN.

- FIND THE AVERAGE RATE OF CHANGE OF HEIGHT W.R.T TIME DURING FIRST 3 SEC.

$$\text{AVERAGE RATE OF CHANGE} = \frac{\text{CHANGE IN HEIGHT}}{\text{CHANGE IN TIME}} = \frac{\Delta y}{\Delta t} = \frac{162-6}{3-0} = \frac{156}{3} = \boxed{52 \text{ FT/SECOND}}$$

DEFN THE AVERAGE RATE OF CHANGE OF HEIGHT WRT TIME IS ALSO CALLED **VELOCITY**.

$$\frac{\text{CHANGE IN HEIGHT}}{\text{CHANGE IN TIME}} = \frac{\text{CHANGE IN DISTANCE}}{\text{CHANGE IN TIME}} = \frac{\Delta y}{\Delta t}$$

- FIND THE AVERAGE VELOCITY OF THE TENNIS BALL FROM $t=4$ TO $t=6$.

USING OUR TABLE OF DATA, WE SEE:

WHEN $t=4$, HEIGHT $= y = 150$ ft, AND WHEN $t=6$, HEIGHT $= y = 30$ ft.

TWO POINTS: $(\underset{t_1}{4}, \underset{y_1}{150})$ & $(\underset{t_2}{6}, \underset{y_2}{30})$

WHY IS A.R.C NEGATIVE? WHAT DOES IT MEAN?

$$\text{AVG RATE OF CHANGE} = \frac{\text{CHANGE IN DISTANCE}}{\text{CHANGE IN TIME}} = \frac{y_2 - y_1}{t_2 - t_1} = \frac{30-150}{6-4} = \boxed{-60 \text{ FT/SECOND}}$$

EX LET $g(x) = \sqrt{x}$. FIND THE AVERAGE RATE OF CHANGE OF $g(x)$ FROM $x=0$ TO $x=3.5$.

WE ARE GIVEN TWO POINTS:

$(0, g(0))$ $(3.5, g(3.5))$

PLUGGING IN $x=0$ TO $g(x)$.. & PLUGGING IN $x=3.5$ TO $g(x)$..

$(\underset{x_1}{0}, \underset{y_1}{0})$ $(\underset{x_2}{3.5}, \underset{y_2}{\sqrt{3.5}})$

WE FOUND THE SLOPE OF THE SECANT LINE PASSING THRU $(0,0)$ & $(3.5, \sqrt{3.5})$

$$\text{AVG RATE OF CHANGE} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\sqrt{3.5} - 0}{3.5 - 0} = \frac{\sqrt{3.5}}{3.5} = \boxed{0.535}$$

EX LET $h(x) = x^2$. FIND THE AVG. RATE OF CHANGE OF $h(x)$ FROM $x=-2$ TO $x=2$.

AGAIN, WE'RE GIVEN 2 POINTS.

$(-2, h(-2))$ & $(2, h(2))$

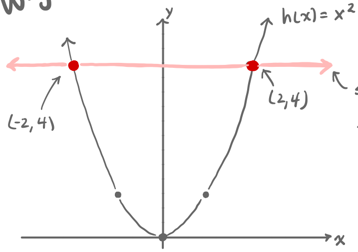
$h(-2) = 4$ & $h(2) = 4$

→ SO WE WANT SLOPE OF LINE BETWEEN THE POINTS:

$(\underset{x_1}{-2}, \underset{y_1}{4})$ & $(\underset{x_2}{2}, \underset{y_2}{4})$

$$\text{AVG RATE OF CHANGE} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-4}{2-(-2)} = \frac{0}{4} = \boxed{0}$$

Why?



SECANT LINE PASSING THRU $(-2,4)$ & $(2,4)$

SECANT LINE IS HORIZONTAL.

⇒ SLOPE OF SECANT LINE IS $\emptyset$

⇒ A.R.C. BETWEEN $(-2, h(-2))$ & $(2, h(2))$ is $\emptyset$.

TOOLS

TO ESTIMATE THE SLOPE BETWEEN TWO POINTS $f(x_1)$ & $f(x_2)$ ON A NONLINEAR FUNCTION $f(x)$, WE CAN FIND THE SLOPE OF THE SECANT LINE PASSING THRU $(x_1, f(x_1))$ & $(x_2, f(x_2))$.

EX MORE EXAMPLES TO TRY..

1. LET $s(x) = \ln(x+1)$. FIND A.R.C. OF $s(x)$ FROM $x=0$ TO $x=5$. IS $s(x)$ INCREASING ON THIS INTERVAL?

2. LET $r(y) = 5^{2y} - 1$. FIND A.R.C. OF $r(y)$ FROM $y=-1$ TO $y=1$

3. LET $f(x) = x^2$. GIVE THE INTERVAL(S) ON WHICH $f(x)$ IS DECREASING.