LINEAR FUNCTIONS

SECTION 1.2

OBJECTIVES:

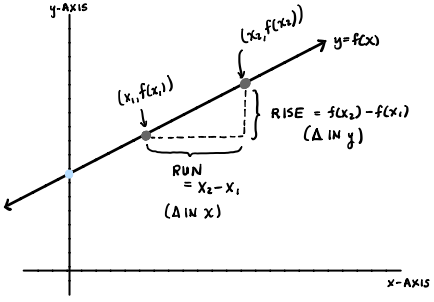
- GIVEN TWO POINTS, WE AIM TO RECALL HOW TO FIND THE EQUATION OF A LINEAR FUNCTION PASSING THRU THE TWO POINTS

DEFINITION WE SAY A FUNCTION f IS **LINEAR** IF THERE EXIST REAL NUMBERS m AND b SUCH THAT f CAN BE WRITTEN AS

$$y = f(x) = mx + b$$

THE NUMBER b IS THE **y-INTERCEPT** OF f & m IS THE **SLOPE** OF f

THE GRAPH OF A LINEAR FUNCTION IS A LINE.



$$\text{SLOPE} = \frac{\text{RISE}}{\text{RUN}} = \frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

THIS IS CALLED THE **DIFFERENCE QUOTIENT**

SLOPE IS THE RATE OF CHANGE OF $y = f(x)$ WITH RESPECT TO x.

DEFINITION LET $f(x) = mx + b$.

- IF $m > 0$, $f(x)$ IS AN **INCREASING FUNCTION**
- IF $m < 0$, $f(x)$ IS A **DECREASING FUNCTION**
- IF $m = 0$, $f(x)$ IS A **CONSTANT FUNCTION** → WHEN WE GRAPH THIS, IT'S JUST A HORIZONTAL LINE

EX PLOT THE FOLLOWING:

- $y = \frac{-1}{2}x - 2$
- $y = -x$
- $y = 5$
- $y = \frac{1}{2}x + 1$

EX

- WRITE THE EQN OF A LINE WITH y-INT 3 THAT IS ALSO DECREASING
- MUST A LINE OF THE FORM $y = mx + b$ HAVE A y-INTERCEPT?

EXAMPLE AT THE PENSACOLA GREYHOUND RACETRACK, RECORD TIMES FOR THE 503 METER RACE DECREASED BY .03 SECONDS EVERY TWO YEARS.

YEAR	1990	1992	1994	1996	1998	
TIME (seconds)	30.18	30.15	30.12	30.09	30.06	

IF y IS THE RECORD TIME (IN SECONDS) & t IS THE # OF YRS SINCE 1990, MAKE A LINEAR FUNCTION THAT MODELS THIS DATA.

$y = f(t) = mt + b$ (t IS YRS SINCE 1990; y IS RECORD TIME (SECONDS))

SO, WE HAVE:

$$y = f(t) = 30.18 - \left(\frac{0.03}{2}\right)t$$

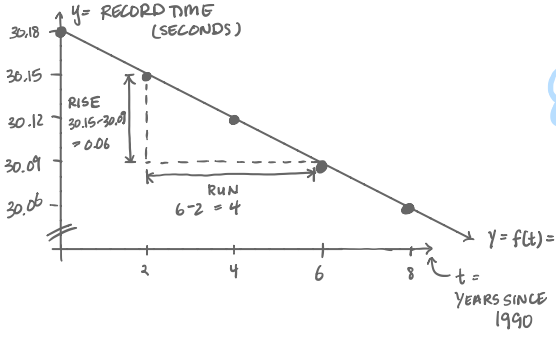
* NOTICE: SINCE y DECREASES AS t INCREASES, $f(t)$ IS A DECREASING FUNCTION.

* THE COEFFICIENT OF t TELLS US THE RATE (IN SECONDS/YR) IN WHICH THE RECORD TIME DECREASES

WE ALSO CALLED THIS THE SLOPE.

$$\text{SLOPE} = \frac{\text{RISE}}{\text{RUN}} = \frac{\Delta y}{\Delta t} = \frac{\Delta \text{ TIME (SEC)}}{\Delta \text{ YEAR}} = \frac{30.15 - 30.18}{1992 - 1990} = \frac{-.03}{2} \text{ SECONDS/YEAR}$$

* THE CONSTANT 30.18 IS THE INITIAL RECORD, OR THE RECORD WHEN $t=0$. (OR, THE RECORD IN 1990)



DOES THIS LINEAR TREND CONTINUE BEYOND 1998??

IN GENERAL: NO!

SUPPOSE THAT IT DOES! THEN, OUR FORMULA PREDICTS THAT THE RECORD TIME IN 2017 OR $t=20$ WILL BE:

$$30.18 - \frac{(0.03)}{2}(20) = 29.88 \text{ SECONDS}$$

which isn't unreasonable!

MORAL: IT CAN BE DANGEROUS TO EXTRAPOLATE DATA TOO FAR INTO THE FUTURE!

BUT THE ACTUAL BEST 503m TIME IN 2017 WAS RECORDED AS 29.93 SECONDS, BY A GREYHOUND NAMED "HEARTBREAK HOTEL"

EX CONSTRUCT LINEAR FORMULAS FOR THE FOLLOWING:

(A) Q AS A FUNCTION OF P

(B) P AS A FUNCTION OF Q

DATA:

P	5	10	15	20
Q	100	90	80	70

(A) "Q AS A FUNCTION OF P" MEANS THAT:

Q = DEPEND. VARIABLE (IT DEPENDS ON p!) → Think of this as y

P = INDEP. VARIABLE → Think of this as x

in $y = mx + b$

m = SLOPE $= \frac{\Delta Q}{\Delta P} = \frac{90-100}{10-5} = \frac{-10}{5} = -2$ $\boxed{m=-2}$

SO WE HAVE NOW:

$Q = -2P + b$ & WE NEED TO SOLVE FOR b.

WE CAN PLUG IN $Q=100$ & $P=5$ TO GET

$100 = -10 + b \longrightarrow \boxed{b = 110}$

→ SO, OUR LINEAR EQN IS:

$\boxed{Q = -2P + 110}$

(B) "P AS A FUNCTION OF Q" MEANS..

P = DEPEND. VARIABLE → Think of this as y

Q = INDEP. VARIABLE → Think of this as x

in $y = mx + b$

m = SLOPE $= \frac{\Delta y}{\Delta x} = \frac{\Delta P}{\Delta Q} = \frac{10-5}{90-100} = \frac{-1}{2}$ $\boxed{m = \frac{-1}{2}}$

SO NOW WE HAVE:

$P = -\frac{1}{2}Q + b$ & WE JUST NEED TO SOLVE FOR b. PLUG IN $Q=100$ & $P=5$..

$5 = -\frac{1}{2}(100) + b$

$= -50 + b \Rightarrow \boxed{b = 55}$

→ $\boxed{P = -\frac{1}{2}Q + 55}$

IN CLASS..

STUDENTS ARE TO COME UP WITH A STEP-BY-STEP PROCEDURE FOR FINDING THE EQN OF A LINE PASSING THRU TWO POINTS