PROPERTIES OF ANTIDERIVATIVES

OBJECTIVES: INVESTIGATE PROPERTIES OF INTEGRATION

RECALL THE FOLLOWING FROM MONDAY:

- TO COMPUTE $\int_a^b f(x)\,dx$ AND $\int f(x)\,dx$, WE NEED TO FIND AN ANTIDERIVATIVE OF $f(x)$.
- WE ALSO FOUND THE FOLLOWING FORMULAS:
  - $\int k\,dx = kx + C$
  - $\int x^n\,dx = \dfrac{x^{n+1}}{n+1} + C$ FOR $n \neq -1$

WE'LL BE FOCUSING ON THE FOLLOWING TODAY:

PROPERTIES OF ANTIDERIVATIVES:

(1) • $\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$

(2) • $\int c \cdot f(x)\,dx = c \cdot \int f(x)\,dx$ (for a constant c)

Why should these be true?

Well.. we know they're true for derivatives!

EX. CONSIDER..

(1) • $\int x^5 + x^7\,dx = \frac{1}{6}x^6 + \frac{1}{8}x^8 + C$

AND $\dfrac{d}{dx}\left(\frac{1}{6}x^6 + \frac{1}{8}x^8 + C\right) = \left(6 \cdot \frac{1}{6}\right)x^5 + \left(8 \cdot \frac{1}{8}\right)x^7 = x^5 + x^7$

(2) • $\int 5x^2\,dx = \frac{5}{3}x^3 + C$

AND $\dfrac{d}{dx}\left(\frac{5}{3}x^3 + C\right) = 3\left(\frac{5}{3}\right)x^2 = 5x^2 = 5\,\dfrac{d}{dx}\left(\frac{1}{3}x^3 + C\right)$

NOTE:

$$\begin{tikzcd} \int f(x)\,dx \arrow[d, shift right, "\text{DIFFERENTIATE}"'] \\ f(x) \arrow[u, shift right, "\text{INTEGRATE}"'] \arrow[d, shift left, "\text{DIFFERENTIATE}"] \\ f'(x) \arrow[u, shift left, "\text{INTEGRATE}"] \end{tikzcd}$$

WE CAN MOVE BETWEEN $\int f(x)\,dx$, $f(x)$, AND $f'(x)$ BY DIFFERENTIATING OR INTEGRATING TO MOVE IN EITHER DIRECTION.

THIS MAKES IT EASIER TO SEE THAT INTEGRATION & DIFFERENTIATION ARE IN SOME SENSE, INVERSE OPERATIONS OF EACHOTHER (ONE UNDOES THE OTHER & VISVERSA)

⇝ YOU CAN CHECK (VIA EXAMPLE) THAT OUR DIFFERENTIATION RULES (CHAIN, PRODUCT, QUOTIENT) DO NOT TRANSLATE NICELY TO INTEGRATION. IN PARTICULAR, WE DONT HAVE A CHAIN OR QUOTIENT RULE FOR INTEGRATION, AND THE ANALOG OF THE PRODUCT RULE IS CALLED INTEGRATION BY PARTS. WE WONT TALK ABOUT THIS BECAUSE IT CAN BECOME REALLY COMPLICATED

SO.. AT THE MOMENT, WE HAVE TWO TOOLS FOR INTEGRATION: THE POWER RULE & THE PROPERTIES ABOVE.

WE ALSO KNOW THE FOLLOWING:

FACT $\int \frac{1}{x}\,dx = \ln|x| + C$

SINCE $d/dx\,(\ln(x)) = 1/x$ ... but if $x<0$, $\ln(x)$ ISNT DEFINED. THEN..

$d/dx\,(\ln(-x)) = \frac{-1}{-x} = 1/x$.

FACT $\int e^x\,dx = e^x + C$

BECAUSE $d/dx\,(e^x + C) = e^x$

Ex. FIND $\int (x+1)^2 dx$ NOTING THAT WE DON'T REALLY HAVE A PRODUCT RULE FOR INTEGRATION

SO... WE'LL HAVE TO SIMPLIFY THIS.

$= (x+1)^2 = (x+1)(x+1) = x^2+2x+1$

SO WE'LL COMPUTE

$$\int x^2+2x+1\, dx$$

$$\int x^2+2x+1\,dx$$

$$= \int x^2\,dx + 2\int x\,dx + \int 1\,dx$$

$$\frac{1}{3}x^3 + 2\left(\frac{1}{2}x^2\right) + x$$

$$= \frac{1}{3}x^3 + x^2 + x + C$$

Ex. $\int_1^3 2x\,dx \longrightarrow$ YOU CAN COMPUTE $\int 2x\,dx$ FIRST

$$\int 2x\,dx = 2\cdot\left(\frac{1}{2}\right)x^2 + C = \underbrace{x^2+C}_{\text{ANTIDERIV}}$$

Recall: Fund. Thm of Calculus

LET $F'(x)=f(x)$. THEN

$$\int_a^b f(x)\,dx = F(b)-F(a)$$

SO.. FUNDAMENTAL THM OF CALCULUS SAYS:

$$\int_1^3 2x\,dx = (3^2+C)-(1^2+C) = (9+C)-(1+C) = 8+C-C = \underline{\underline{8}}$$

Ex. $\int_1^4 \frac{1}{\sqrt{x}}\,dx = \int_1^4 x^{-1/2}\,dx$ $\longrightarrow n = -1/2$, $n+1 = \frac{-1}{2}+\frac{2}{2} = \frac{1}{2}$

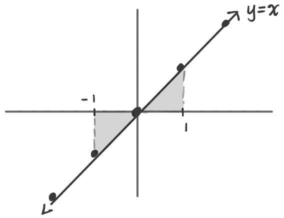
$\int x^{-1/2}\,dx = 2x^{1/2}+C$. THEN...

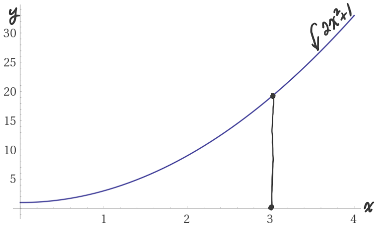
$$\int_1^4 x^{-1/2}\,dx = \left(2(4)^{1/2}+C\right) - \left(2(1)^{1/2}+C\right)$$

Ex FIND SOME NONCONSTANT FUNCTION $f(x)$ AND NUMBERS a,b SO THAT

$$\int_a^b f(x)\,dx = 0 \longrightarrow$$ GEOMETRICALLY, WHAT'S GOING ON HERE?

AN EXAMPLE OF THIS:

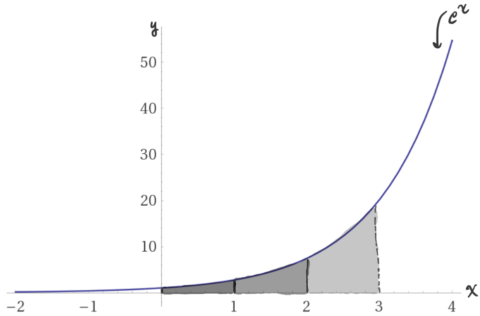




Ex. COMPUTE $\int_3^3 2x^2+1\,dx$. GEOMETRICALLY, WHAT'S HAPPENING HERE?

ANTIDERIVATIVE FOR $2x^2+1 \longrightarrow \frac{2x^3}{3}+x+C$

$$\int_3^3 2x^2+1\,dx = \frac{2x^3}{3}+x+C\Big|_3^3 = \underline{\underline{0}}$$



Ex. COMPUTE THE FOLLOWING:

- $\int_0^1 e^x\,dx = e^x\Big|_0^1 = e^1 - e^0 = e-1$
- $\int_1^2 e^x\,dx = e^x\Big|_1^2 = e^2-e^1$
- $\int_2^3 e^x\,dx = e^x\Big|_2^3 = e^3-e^2$
- $\int_0^3 e^x\,dx =$

Ex COMPUTE THE FOLLOWING:

- $\int_{-4}^{-2} x^3+x\,dx$
- $\int_{-2}^{0} x^3+x\,dx$
- $\int_{0}^{2} x^3+x\,dx$
- $\int_{2}^{3} x^3+x\,dx$
- $\int_{-4}^{3} x^3+x\,dx$