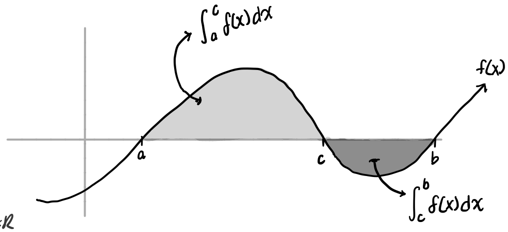
DEFINITE & INDEFINITE INTEGRALS

OBJECTIVES

INVESTIGATE METHODS OF COMPUTING DEFINITE INTEGRALS VIA FUND. THM. OF CALC & INDEFINITE INTEGRALS



Recall from friday...

Def WHEN $f(x)$ IS POSITIVE AND $a<b$, THE AREA UNDER THE GRAPH OF f FROM $x=a$ TO $x=b$ IS:

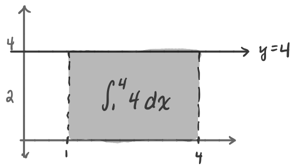
$$\int_a^b f(x)\,dx.$$

THIS IS CALLED A **DEFINITE INTEGRAL**

WE ALSO SAW THAT (ATLEAST IN THIS CLASS)..

ANTIDIFFERENTIATION = "UNDOING" DERIVATIVE
‖
INTEGRATING

Ex. FIND THE AREA UNDER THE CURVE $y=4$ FROM $x=1$ to $x=4$.



WRITING $\int_1^4 4\,dx$ MEANS THE SAME THING

AREA OF RECTANGLE TO THE LEFT IS $(4-1)\cdot 4 = 12$

SO $\int_1^4 4\,dx = 12$.

WHAT IS A FUNCTION $f(x)$ SO THAT $f'(x)=4$? $4x$? $4x+3$? $4x-1$?

IN GENERAL... $f(x) = 4x + c$, FOR A CONSTANT c.

NOTICE THE FOLLOWING:

$$\int_1^4 4\,dx = 12 = 4\cdot(4) - 4(1)$$

THEOREM (THE FUNDAMENTAL THEOREM OF CALCULUS)

IF $F'(t)$ IS CONTINUOUS FROM $t=a$ to $t=b$, THEN

$$\int_a^b F'(t)\,dt = \underbrace{F(b) - F(a)}_{\text{ALSO WRITTEN AS } F(t)\big|_a^b}$$

Def. LET $F(x)$ BE AN ANTIDERIVATIVE OF $f(x)$. ($F'(x) = f(x)$). THEN, $F(x)+c$ (WITH c A CONSTANT) IS THE FAMILY OF ANTIDERIVATIVES OF $f(x)$. WE SAY

$$\int f(x)\,dx = F(x)+c$$

THE **INDEFINITE INTEGRAL** OF $f(x)$.

NOTICE THE FOLLOWING:

$$\underbrace{\int_a^b f(x)\,dx}_{\text{THIS IS A NUMBER. (EXACTLY ONE)}} \qquad \underbrace{\int f(x)\,dx}_{\text{THIS IS A FAMILY OF FUNCTIONS. (INFINITELY MANY)}}$$

TO EVALUATE $\int_a^b f(x)\,dx$ AND $\int f(x)\,dx$, WE NEED TO KNOW $F(x)$ - AN ANTIDERIVATIVE OF $f(x)$.

SO... HOW CAN WE FIND $F(x)$ GIVEN $f(x)$?

THERE ARE SOME FUNCTIONS, LIKE $f(x) = e^{-x^2}$ WHERE NO SUCH "NICE" ANTIDERIVATIVE EXISTS.

FROM THE FIRST EXAMPLE, WE KNOW $\int 4\,dx = 4x + C$.

IN GENERAL...

FACT.

K IS A CONSTANT.

$$\int k\,dx = kx + C$$

QUESTION WHAT ABOUT $\int x\,dx$? OR $\int x^2\,dx$? CAN WE COME UP WITH A FORMULA TO FIND $\int x^n\,dx$?

WHAT CAN I TAKE THE DERIVATIVE OF TO GET x? x^2? x^n?