MORE u-SUBSTITUTION

OBJECTIVES. • PRACTICE u-SUB ON DEFINITE INTEGRALS
• CONTINUE TO PRACTICE u-SUB ON INDEF. INTEGRALS

THE GENERAL STRATEGY I USE IS TO FIND THE ANTIDERIV. FIRST... THEN WORRY ABOUT LIMITS OF INTEGRATION LAST

Ex. $\int_0^1 2xe^{x^2+1}\,dx$

SO WE'LL USE u-SUB ON:

$$\int 2xe^{x^2+1}\,dx$$

$u = x^2+1 \longrightarrow \int 2xe^u\,dx$

$du = 2x\,dx \longrightarrow \int e^u\,du$

NOW WE HAVE

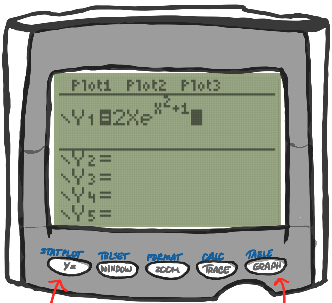
$\int e^u\,du = e^u + C \rightarrow e^{x^2+1} + C$ ... SO WE'VE FOUND THE (FAMILY OF) ANTIDERIVATIVE.

NOW, WE'LL DEAL WITH THE LIMITS OF INTEGRATION.

$$\int_0^1 2xe^{x^2+1}\,dx = e^{x^2+1}+C\Big|_0^1 = \left(e^{1^2+1}+C\right) - \left(e^{0^2+1}+C\right)$$

$$= e^2 + C - (e + C) = \boxed{e^2 - e}$$

LET'S TRY THIS WITH A CALCULATOR.

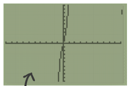


STEP 1

STEP 1.5

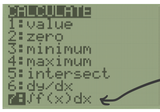
STEP 1: PRESS Y= & ENTER THE FUNCTION YOU WANT TO INTEGRATE.

STEP 1.5: PRESS THE GRAPH BUTTON TO GRAPH THE FUNCTION.



THE GRAPH SHOULD LOOK LIKE THIS.

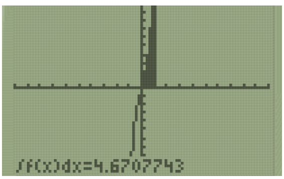
STEP 2 PRESS 2ND + TRACE AND SCROLL DOWN TO OPTION #7:



INTEGRATE!

STEP 3 TYPE THE LOWER LIMIT OF INTEGRATION, THEN PRESS ENTER

TYPE THE UPPER LIMIT OF INTEGRATION, PRESS ENTER AGAIN



← THIS IS WHAT YOU SHOULD END UP WITH.

NOTICE $e^2 - e = 4.67\ldots$

SO OUR ANSWER ABOVE AGREES WITH THE CALCULATOR

Ex. $\int x\sqrt[5]{x+3}\,dx$

$u = x+3 \longrightarrow \int x\sqrt[5]{u}\,dx$

$du = 1\,dx \longrightarrow \int x\sqrt[5]{u}\,du$

$\longrightarrow$ ISSUE: THERE'S AN x LEFT IN OUR INTEGRAL.

SOMEHOW, WE NEED TO REWRITE x IN TERMS OF u.

NOTICE: WE HAD $u = x+3$, SO WE ALSO MUST HAVE $u-3 = x$!

NOW WE HAVE:

DISTRIBUTE

$$\int (u-3)\sqrt[5]{u}\,du = \int (u-3)u^{1/5}\,du$$

$$= \int u^{6/5} - 3u^{1/5}\,du = \frac{u^{11/5}}{11/5} - \frac{3u^{6/5}}{6/5} + C = \frac{5u^{11/5}}{11} - \frac{15u^{6/5}}{6} + C$$

$$= \boxed{\frac{5(x+3)^{11/5}}{11} - \frac{15(x+3)^{6/5}}{6} + C}$$

Ex $\int \frac{x^2}{x^3+1}\,dx$

$u = x^3+1 \longrightarrow \int \frac{x^2}{u}\,dx$

$du = 3x^2\,dx \longrightarrow$ WE ONLY HAVE x^2dx IN THE INTEGRAL

$\frac{1}{3}du = x^2\,dx$

so our integral becomes: ...

$$\int \frac{1}{u}\frac{1}{3}du = \frac{1}{3}\int \frac{1}{u}\,du = \frac{1}{3}\ln|u| + C$$

$$= \boxed{\frac{1}{3}\ln|x^3+1| + C}$$

Ex. $\int 2x^3\sqrt{x^2+1}\,dx$

$u = x^2+1 \longrightarrow \int 2x^3\sqrt{u}\,dx$

$du = 2x\,dx$

$\hookrightarrow$ Notice: $2x^3 = 2x(x^2)$ so... OUR INTEGRAL BECOMES:

$$\int x^2\sqrt{u}\,du$$

NOW: $u = x^2+1 \Rightarrow u-1 = x^2$. WE CAN NOW WRITE OUR INTEGRAL ENTIRELY IN TERMS OF u.

$$\int (u-1)u^{1/2}\,du$$

... AND PROCEED AS USUAL.

Ex. $\int e^{t+e^t}\,dt$

[Caution: YOU MIGHT TRY TO LET $u = t+e^t$. WHY DOESN'T THIS WORK?]

NOTICE THE FOLLOWING:

$$\int e^{t+e^t}\,dt = \int e^t\cdot e^{e^t}\,dt.$$

NOW WE CAN LET $u = e^t$, SO $du = e^t\,dt$.

$$= \int e^u\,du = e^u + C$$

$$= \boxed{e^{e^t} + C}$$