

MAXIMA, MINIMA & CONCAVITY

SECTIONS 4.1 & 4.2
IN THE TEXT

OBJECTIVES:

- PRACTICE FINDING LOCAL MINS/MAXS
- DEFINE CONCAVITY, & DEVISE A METHOD OF DETERMINING WHEN $f(x)$ IS CONCAVE UP/DOWN

Ex. (CONTINUED FROM FRIDAY)

ON FRIDAY WE STARTED TO FIND LOCAL MAXIMA/MINIMA OF $f(x) = x^5(2-x)^6$

RECALL OUR PROCESS:

STEP 1 → FIND $f'(x)$.

IN THIS CASE WE FOUND $f'(x) = -6x^5(2-x)^5 + 5x^4(2-x)^6$

STEP 2 → SET $f'(x) = 0$ AND SOLVE FOR x TO FIND **CRITICAL POINTS**.

$$-6x^5(2-x)^5 + 5x^4(2-x)^6 = 0 \rightarrow \text{WHAT CAN I FACTOR OUT OF THIS?}$$

$$(2-x)^5 [-6x^5 + 5x^4(2-x)] = 0 \rightarrow \text{WHAT ELSE CAN I FACTOR?}$$

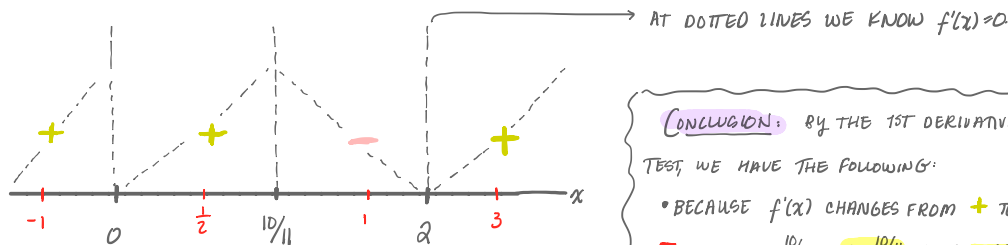
$$x^4(2-x)^5 [-6x + 10 - 5x] = 0 \xrightarrow{\text{REWRITE}} x^4(2-x)^5 [-11x + 10] = 0$$

NDW I CAN SET EACH FACTOR TO ZERO.

THIS GIVES $x=0$, $x=2$, $x=10/11$ AS CRITICAL POINTS

STEP 3 DETERMINE BEHAVIOR OF $f'(x)$ AROUND CRITICAL POINTS.

(THIS IS WHY WE WERE USING A NUMBER LINE ON FRIDAY)



CONCLUSION: BY THE 1ST DERIVATIVE

TEST, WE HAVE THE FOLLOWING:

• BECAUSE $f'(x)$ CHANGES FROM + TO -

AROUND $x=10/11$, $x=10/11$ IS A **LOCAL**

MAXIMUM OF $f(x)$

• BECAUSE $f'(x)$ CHANGES FROM - TO +

AROUND $x=2$, $x=2$ IS A **LOCAL**

MINIMUM OF $f(x)$.

$$f'(-1) = (-1)^4(2-(-1))^5[-11(-1)+10] > 0 \rightarrow f(x) \text{ INCREASING}$$

$$f'(1/2) = (1/2)^4(2-(1/2))^5[-11(1/2)+10] > 0 \rightarrow f(x) \text{ INCREASING}$$

$$f'(1) = (1)^4(2-1)^5[-11+10] < 0 \rightarrow f(x) \text{ DECREASING}$$

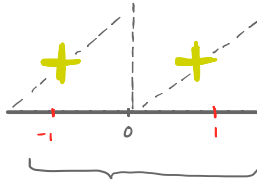
$$f'(3) = (3)^4(2-3)^5[-11(3)+10] > 0 \rightarrow f(x) \text{ INCREASING}$$

Example FIND LOCAL MAXIMA/MINIMA OF $f(x) = x^3$

STEP 1 $f'(x) = 3x^2$

STEP 2 SET $3x^2 = 0$, SOLVE FOR x . WE JUST GET $x=0$ HERE, SO $x=0$ IS OUR ONLY CRITICAL POINT.

STEP 3 DETERMINE BEHAVIOR OF $f'(x)$ AROUND CRITICAL POINT.



TEST POINTS: -1 AND 1..

$$f'(-1) = 3(-1)^2 = \text{POSITIVE}$$

$\Rightarrow f(x)$ INCREASING

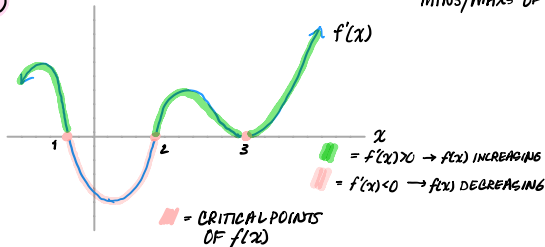
$$f'(1) = 3(1) = \text{POSITIVE}$$

$\Rightarrow f(x)$ INCREASING

BY THE FIRST DERIVATIVE TEST, $f(x) = x^3$ HAS NO LOCAL MAX/MIN.

Ex USING THE FOLLOWING GRAPHS, DETERMINE THE CRITICAL POINTS OF $f(x)$, AND STATE IF THEY ARE LOCAL

(A)



mins/maxs of $f(x)$.

AROUND CRITICAL POINT #1...

$f'(x)$ GOES FROM + TO -

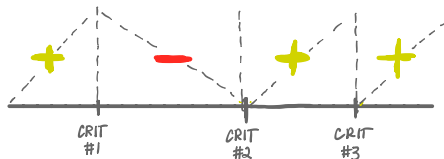
$\Rightarrow$ CRITICAL POINT #1 IS A
LOCAL MAX OF $f(x)$

AROUND CRITICAL POINT #2...

$f'(x)$ GOES FROM - TO +

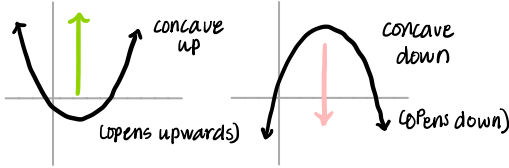
$\Rightarrow$ CRITICAL POINT #2 IS A
LOCAL MIN OF $f(x)$.

IN TERMS OF THE # LINES WE'VE BEEN DRAWING...



BY THE FIRST DERIV. TEST
 $f(x)$ HAS LOCAL MAX @
CRIT #1 & LOCAL MIN
@ CRIT #2

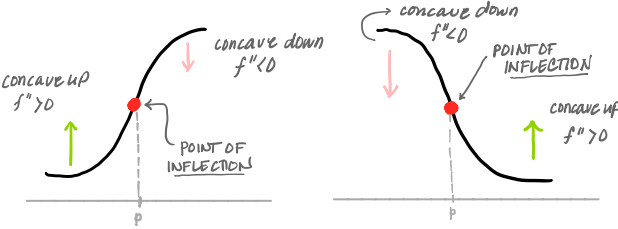
DEFINITION A POINT $x=p$ WHERE THE GRAPH OF A FUNCTION $f(x)$ CHANGES CONCAVITY IS CALLED AN **INFLECTION POINT** OF $f(x)$.



AN **INFLECTION POINT** IS ALSO WHERE $f'(x)$ CHANGES FROM INCREASING TO DECREASING OR FROM DECREASING TO INCREASING.

(IN SOME SENSE, IT'S LIKE A CRITICAL POINT OF $f'(x)$).

THINK: HOW DOES THIS DIFFER FROM A CRITICAL POINT?



IDEA:

TO FIND INFLECTION POINTS, WE CAN FOLLOW THE SAME PROCEDURE THAT WE USED FOR THE FIRST DERIVATIVE TEST... EXCEPT WE'LL USE THE SECOND DERIVATIVE INSTEAD.

EX FIND INFLECTION POINTS OF $f(x) = x^3 - 9x^2 - 48x + 52$.

STEP 1 FIND $f''(x)$.

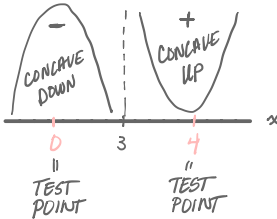
IF $f(x) = x^3 - 9x^2 - 48x + 52$... THEN $f'(x) = 3x^2 - 18x - 48$ AND $f''(x) = 6x - 18$

STEP 2 SET $f''(x) = 0$ & SOLVE FOR x

$$f''(x) = 6x - 18 = 0 \rightsquigarrow 6x = 18 \rightsquigarrow x = 3$$

POSSIBLE INFLECTION POINT

STEP 3 DETERMINE BEHAVIOR OF $f''(x)$ AROUND $x=3$.



• $f''(0) = 6(0) - 18 = -18 < 0$.

so $f''(x) < 0 \Rightarrow f(x)$ CONCAVE DOWN

• $f''(4) = 6(4) - 18 > 0$

so $f''(x) > 0 \Rightarrow f(x)$ CONCAVE UP

THUS, SINCE $f''(x)$ CHANGES SIGN AROUND $x=3$, $x=3$ IS THE ONLY INFLECTION POINT OF $f(x)$.